

MATHEMATICS

Stage 5

Measurement and Space

Area and Surface Area

Volume

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Exam Equivalent Time: 136.5 minutes (based on allocation ~1.25 min/mark)

Worked Solutions

1. **C** Volume (002142)

Convert all measurements to centimetres:

$$\begin{aligned} V &= l \times b \times h \\ &= 30 \times 5 \times 400 \\ &= 60\,000 \text{ cm}^3 \\ &\Rightarrow C \end{aligned}$$

2. **C** Volume (029073)

$$\begin{aligned} V &= \frac{1}{3}Ah \\ A &= 35 \times 35 \\ &= 1225 \text{ m}^2 \\ \therefore V &= \frac{1}{3} \times 1225 \times 22 \\ &= 8983.33 \dots \text{ m}^3 \\ &\Rightarrow C \end{aligned}$$

3. **B+** Volume (060327)

$$1 \text{ mL} = 1 \text{ cm}^3 \Rightarrow 1 \text{ L} = 1000 \text{ cm}^3$$

$$\begin{aligned} \text{Volume} &= Ah \\ 12\,000 &= 240 \times h \\ h &= \frac{12\,000}{240} \\ &= 50 \text{ cm} \end{aligned}$$

$$\Rightarrow D$$

Mean mark 35%.

4. **B+** Area and Surface Area (010181)

Total Area = Area of cylinder + ½ sphere

Area of cylinder = $2\pi rh$

$= 2\pi \times 24 \times 30$

$= 4523.9$

Area of ½ sphere = $\frac{1}{2} \times 4\pi r^2$

$= \frac{1}{2} \times 4\pi \times 24^2$

$= 3619.1$

∴ Total area = $4523.9 + 3619.1$

$= 8143 \text{ m}^2$

⇒ A

5. **E** Area and Surface Area (245991)

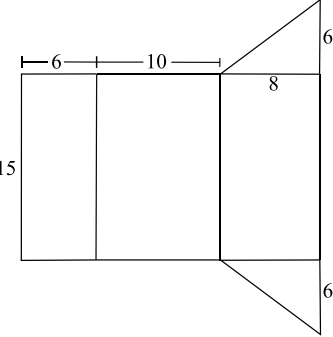
S.A. = $2 \times (6 \times 14) + 2 \times (6 \times 5) + 2 \times (5 \times 14)$

$= 2(84 + 30 + 70)$

$= 368 \text{ m}^2$

6. **D** Area and Surface Area (245997)

i.



ii. S.A. = $(15 \times 6) + (15 \times 10) + (15 \times 8) + 2 \times (\frac{1}{2} \times 8 \times 6)$

$= 90 + 150 + 120 + 2(24)$

$= 408 \text{ cm}^2$

7. **D** Area and Surface Area (246012)

Area (front side) = $(4 \times 2) + (2 \times 2) = 12 \text{ m}^2$

S.A. = $2 \times 12 + 2 \times (5 \times 4) + 4 \times (5 \times 2)$

$= 24 + 2(20) + 4(10)$

$= 104 \text{ m}^2$

8. **D** Volume (247440)

Volume = $\pi r^2 h$

$= \pi \times 9^2 \times 45$

$= 11\,451.10\dots$

$= 11\,451 \text{ cm}^3 \text{ (nearest cm}^3\text{)}$

9. **D** Volume (247252)

Volume = $l \times b \times h$

$= 9.5 \times 4.2 \times 6.8$

$= 271.32$

$= 271 \text{ cm}^3 \text{ (nearest cm}^3\text{)}$

10. **C** Area and Surface Area (060377)

SA = $4\pi r^2$

$= 4 \times \pi \times 5^2$

$= 314.15\dots$

$= 314 \text{ cm}^2 \text{ (nearest cm}^2\text{)}$

11. **C** Volume (247456)

Volume = $\frac{1}{3} \times A \times h$

$= \frac{1}{3} \times 120 \times 120 \times 65$

$= 312\,000 \text{ m}^3$

12. C Volume (247473)

$$\begin{aligned}\text{Volume} &= \frac{1}{3} \times A \times h \\ &= \frac{1}{3} \times \pi \times 9.5^2 \times 19.5 \\ &= 1842.936\dots \\ &= 1842.94 \text{ cm}^3\end{aligned}$$

13. C Volume (247256)

$$\begin{aligned}\text{Area (trapezium)} &= \frac{1}{2}h(a + b) \\ &= \frac{1}{2} \times 4(10 + 14) \\ &= 48 \text{ m}^2\end{aligned}$$

$$\begin{aligned}\text{Volume} &= A \times h \\ &= 48 \times 9 \\ &= 432 \text{ m}^3\end{aligned}$$

14. C Volume (247452)

$$\begin{aligned}\text{Area (face)} &= (9 \times 13) + \left(\frac{1}{2} \times 13 \times 7\right) \\ &= 162.5 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Volume} &= Ah \\ &= 162.5 \times 6 \\ &= 975 \text{ cm}^3\end{aligned}$$

15. C Volume (247434)

$$\begin{aligned}\text{Area (triangle)} &= \frac{1}{2} \times b \times h \\ &= \frac{1}{2} \times 9 \times 12 \\ &= 54 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Volume} &= A \times h \\ &= 54 \times 7 \\ &= 378 \text{ cm}^3\end{aligned}$$

16. C Volume (247175)

$$\text{Radius} = \frac{23}{2} = 11.5 \text{ cm}$$

$$\begin{aligned}\text{Volume} &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3} \times \pi \times 11.5^3 \\ &= 6370.626\dots \\ &= 6370.6 \text{ cm}^3 \text{ (to 1 d.p.)}\end{aligned}$$

17. C Area and Surface Area (246181)

$$\begin{aligned}\text{Surface Area} &= 2\pi r^2 + 2\pi rh \\ &= 2\pi(1.6)^2 + 2\pi \times 1.6 \times 4.2 \\ &= 58.307\dots \\ &= 58.31 \text{ m}^2 \text{ (2 d.p.)}\end{aligned}$$

18. C Area and Surface Area (246349)

$$\begin{aligned}\text{SA} &= (10 \times 10) + 4 \times \left(\frac{1}{2} \times 10 \times 15\right) \\ &= 100 + 4(75) \\ &= 400 \text{ cm}^2\end{aligned}$$

19. **C** Area and Surface Area (157659)

S.A. of 1 face of cube = $8 \times 8 = 64 \text{ cm}^2$

Height of triangle = $11 - 8 = 3 \text{ cm}$

$$\begin{aligned} \therefore \text{S.A. (triangular prism)} &= 2 \times \left(\frac{1}{2} \times 8 \times 3 \right) + 2 \times (5 \times 8) \\ &= 24 + 80 \\ &= 104 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \therefore \text{Total S.A.} &= 5 \times 64 + 104 \\ &= 424 \text{ cm}^2 \end{aligned}$$

20. **B** Area and Surface Area (246057)

One strategy:

Calculate the surface area (S.A.) of the 2 objects joined separately and deduct the area where they join.

S.A. (large cube) = $6 \times 6 \times 6 = 216 \text{ cm}^2$

S.A. (2 smaller cubes joined together)

$$= 2 \times (3 \times 3) + 4 \times (6 \times 3)$$

$$= 90 \text{ cm}^2$$

S.A. to be deducted (where cubes are attached)

$$= 2 \times (6 \times 3)$$

$$= 36 \text{ cm}^2$$

$$\begin{aligned} \therefore \text{S.A. (new object)} &= 216 + 90 - 36 \\ &= 270 \text{ cm}^2 \end{aligned}$$

21. **B** Area and Surface Area (118400)

Flatten out the half cylinder,

$$\begin{aligned} \text{Width} &= \frac{1}{2} \times \text{circumference} \\ &= \frac{1}{2} \times \pi \times 3.8 \\ &= 5.969 \dots \end{aligned}$$

$$\begin{aligned} \therefore \text{Sheeting required} &= 10 \times 5.969 \dots \\ &= 59.69 \dots \\ &= 60 \text{ m}^2 \text{ (nearest m}^2\text{)} \end{aligned}$$

22. **B** Volume (247465)

Volume (cube) = $5 \times 5 \times 5$

$$= 125 \text{ m}^3$$

Volume (pyramid) = $\frac{1}{3}Ah$

$$= \frac{1}{3} \times 5 \times 5 \times 3$$

$$= 25 \text{ cm}^3$$

$$\text{Total volume} = 125 + 25 = 150 \text{ m}^3$$

23. B Volume (026191)

i. $V = l \times b \times h$

$$= 21 \times 8 \times 9$$

$$= 1512 \text{ cm}^3$$

ii. Volume of each hole

$$= \pi r^2 h$$

$$= \pi \times 1.4^2 \times 8$$

$$= 49.260 \dots \text{ cm}^3$$

$\therefore$ Volume of clay still in brick

$$= 1512 - (3 \times 49.260 \dots)$$

$$= 1364.219 \dots$$

$$= 1364 \text{ cm}^3 \text{ (nearest whole)}$$

iii. Percentage of clay removed

$$= \frac{(3 \times 49.260 \dots)}{1512} \times 100$$

$$= 9.773 \dots$$

$$= 9.8\% \text{ (1 d.p.)}$$

24. B Area and Surface Area (246337)

i. Let s = slant height

$$s^2 = 6^2 + 8^2$$

$$= 100$$

$$s = 10 \text{ m}$$

ii. $SA = (12 \times 12) + 4 \times (\frac{1}{2} \times 12 \times 10)$

$$= 144 + 4(60)$$

$$= 384 \text{ m}^2$$

25. B Area and Surface Area (246092)

$$\text{Total Area} = (2 \times 12) + 2 \times (3 \times 5) + (8 \times 3)$$

$$= 78 \text{ cm}^2$$

26. B Area and Surface Area (246199)

$$\text{Surface Area (half cylinder)} = \frac{1}{2} (2\pi r^2 + 2\pi r h)$$

$$= \frac{1}{2} (2\pi \times 17.5^2 + 2\pi \times 17.5 \times 40)$$

$$= 3161.227 \dots \text{ cm}^2$$

$$\text{Area (rest)} = (35 \times 40) + 2(35 \times 30) + 2(40 \times 30)$$

$$= 1400 + 2(1050) + 2(1200)$$

$$= 5900 \text{ cm}^2$$

$$SA \text{ (total)} = 3161 + 5900 = 9061 \text{ cm}^2$$

27. B+ Volume (246665)

The face of the pool is a rectangle and trapezium (composite).

$$\text{Area (face)} = (2.3 \times 4) + \frac{1}{2} h(a + b)$$

$$= 9.2 + \frac{1}{2} \times 8(2.3 + 1.1)$$

$$= 22.8 \text{ m}^2$$

$$\text{Volume} = Ah$$

$$= 22.8 \times 6$$

$$= 136.8 \text{ m}^3$$

28. B+ Volume (246626)

Volume = Ah where A is the area of a trapezium

$$A = \frac{1}{2} h(a + b)$$

$$= \frac{1}{2} \times 8(11 + 5)$$

$$= 64 \text{ cm}^2$$

$$\therefore V = Ah = 64 \times 9 = 576 \text{ cm}^3$$

COMMENT: Note that h in the "area" formula is different to the h used in the "volume" formula.

29. B+ Volume (247079)

Since diameter sphere = 12 cm
⇒ Radius of cylinder = 6 cm
Height of cylinder = 2 × diameter of sphere
= 2 × 12
= 24 cm

∴ Volume cylinder = $\pi r^2 h$
= $\pi \times 6^2 \times 24$
= 2714.336...
= 2714 cm³

30. B+ Volume (118521)

Converting megalitres to m³ (using 1 m³ = 1000 L):

1.26 ML = $\frac{1.26 \times 10^6}{10^3}$
= 1.26 × 10³ m³
= 1260 m³

$V = \pi r^2 h$
1260 = $\pi \times 9^2 \times h$
 $h = \frac{1260}{\pi \times 9^2}$
= 4.951...
= 4.95 m (2 d.p.)

31. A Volume (247101)

Volume = Area of annulus × h
= $(\pi R^2 - \pi r^2) \times 2.8$
= $(\pi \times 0.45^2 - \pi \times 0.35^2) \times 2.8$
= 0.7037...
= 0.70 m³

32. A Area and Surface Area (246160)

Surface area of 1 face of the cube
= $\frac{150}{6}$
= 25 cm²

Let x = length of 1 side
 $x^2 = 25$
∴ $x = 5$ cm

33. A Area and Surface Area (246356)

$r = 6$ cm, $l = 10$ cm

SA = $\pi r^2 + \pi rl$
= $\pi \times 6^2 + \pi \times 6 \times 10$
= 301.592...
= 301.59 cm² (2 d.p.)

34. A Area and Surface Area (246166)

Let w = width of the brick

Surface Area = $2 \times [8w + 22w + (8 \times 22)]$
982 = $2(30w + 176)$
982 = $60w + 352$
60w = 630
∴ $w = 10.5$ cm

Mean mark 48%.

35. A Area and Surface Area (195581)

S.A. of Cylinder $= \pi r^2 + 2\pi rh$
 $= \pi(2^2) + 2\pi(2)(3)$
 $= 16\pi \text{ cm}^2$

Mean mark 50%.

S.A. of Hemisphere $= \frac{1}{2} \times 4\pi r^2$
 $= 2\pi(3^2)$
 $= 18\pi \text{ cm}^2$

Area of Annulus $= \pi R^2 - \pi r^2$
 $= \pi(3^2) - \pi(2^2)$
 $= 5\pi \text{ cm}^2$

Total S.A. $= 16\pi + 18\pi + 5\pi$
 $= 39\pi$
 $= 122.522\dots$
 $= 122.5 \text{ cm}^2 \text{ (to 1 d.p.)}$

36. A Volume (247488)

Radius (original cone) $= 12 \text{ cm}$
Radius (small cone) $= 6 \text{ cm}$
Volume $= \text{Original cone} - \text{Small cone}$
 $= \left(\frac{1}{3} \times \pi \times 12^2 \times 60\right) - \left(\frac{1}{3} \times \pi \times 6^2 \times 30\right)$
 $= 7916.81\dots$
 $= 7916 \text{ cm}^3 \text{ (nearest cm}^3\text{)}$

37. A+ Area and Surface Area (246153)

S.A. of rectangular prism
 $= 2 \times (h \times h) + 4 \times (2h \times h)$
 $= 2h^2 + 8h^2$
 $= 10h^2$

S.A. of cube $= 6 \times (h \times h) = 6h^2$

$\therefore$ Ratio of rectangular prism to cube
 $= 10h^2 : 6h^2$
 $= 5 : 3$