

MATHEMATICS

Stage 5

Number and Algebra, Equations

Quadratics and Cubics

Number and Algebra, Financial Mathematics

Compound Interest

Simple Interest

Depreciation

Earning and Spending Money

Number and Algebra, Linear Relationships

Cartesian Plane

Measurement and Space, Trigonometry

Right-Angled Trig

Teacher: Kyall Duncan

Exam Equivalent Time: 91.5 minutes (based on allocation ~1.25 min/mark)



Worked Solutions

1. **C** Cartesian Plane (196472)

By elimination:

y -intercept = 2 $\rightarrow$ Eliminate B and C

Gradient is negative $\rightarrow$ Eliminate D

$\Rightarrow A$

2. **C** Compound Interest (137253)

$$FV = PV(1 + r)^n$$

$$= 1000(1 + 0.035)^2$$

$$= \$1071.23$$

$\Rightarrow B$

3. **C** Cartesian Plane (228155)

Coordinates of C are:

$$(7 - 4, 5 - 3) = (3, 2)$$

$\Rightarrow A$

4. **B** Quadratics and Cubics (097166)

$$3x^2 - x - 2 = (3x + 2)(x - 1)$$

$\Rightarrow D$

5. **B** Simple Interest (253265)

$$1 \text{ year } 8 \text{ months} = 12 + 8 = 20 \text{ months}$$

$$\text{Interest} = Prn$$

$$= 10\,000 \times \frac{4.25}{100} \times \frac{20}{12}$$

$$= \$708$$

$$\therefore \text{Value of Investment} = 10\,000 + 708 = \$10\,708$$

$\Rightarrow C$

6. **B** Simple Interest (001958)

$$I = Prn = 1000 \times \frac{3.75}{100} \times \frac{15}{12} = \$46.88$$

$$\therefore \text{Investment worth} = 1000 + 46.88 = \$1046.88$$

$\Rightarrow A$

7.

B

 Depreciation (254524)

$$r = 20\% = 0.2$$

Test the values of given options using: $S = V_0(1 - r)^n$

$$S(\text{Jan20}) = 5000(1 - 0.2)^3 = 5000 \times 0.8^3 = \$2560$$
$$S(\text{Jan22}) = 5000 \times 0.8^5 = \$1638.40$$
$$S(\text{Jan23}) = 5000 \times 0.8^6 = \$1310.72$$
$$\Rightarrow D$$

8.

B

 Simple Interest (253284)

Total interest paid = $2350 - 2000 = \$350$

$$I = Prn$$
$$350 = 2000 \times r \times 1$$
$$r = \frac{350}{2000}$$
$$= 0.1750$$
$$= 17.50\%$$
$$\Rightarrow B$$

9.

B

 Simple Interest (253343)

$$I = 27\,000, \quad r = \frac{8}{100}, \quad n = 6$$
$$I = Prn$$
$$27\,000 = \frac{P \times 8 \times 6}{100}$$
$$P = \frac{2\,700\,000}{48}$$
$$= \$56\,250$$
$$\Rightarrow C$$

10.

B

 Depreciation (010131)

$$S = V_0(1 - r)^n$$
$$= 19\,990\left(1 - \frac{18}{100}\right)^3$$
$$= 19\,990(0.82)^3$$
$$= \$11\,021.85$$
$$\Rightarrow C$$

11.

B+

 Earning and Spending Money (015088)

$$\begin{aligned} \text{Salary (4 weeks)} &= 4 \times 280 \\ &= \$1120 \end{aligned}$$
$$\begin{aligned} \text{Holiday loading} &= 1120 \times 17.5\% \\ &= \$196 \end{aligned}$$
$$\begin{aligned} \therefore \text{Total pay} &= 1120 + 196 \\ &= \$1316 \end{aligned}$$
$$\Rightarrow D$$

12.

B+

 Earning and Spending Money (014181)

$$\begin{aligned} \text{Amount earned last week} &= 12 \times 1.5 \times 15 \\ &= \$270 \end{aligned}$$
$$\begin{aligned} \text{Double time rate} &= 2 \times 15 \\ &= \$30/\text{hr} \end{aligned}$$
$$\begin{aligned} \therefore \# \text{ Hours at double time} &= \frac{270}{30} \\ &= 9 \text{ hrs} \end{aligned}$$
$$\Rightarrow D$$

13.

A

 Compound Interest (029229)

Using $FV = PV(1 + r)^n$

$$r = \frac{4\%}{4} = 1\% = 0.01 \text{ per quarter}$$
$$n = 5 \times 4 = 20 \text{ quarters}$$
$$60\,000 = PV(1 + 0.01)^{20}$$
$$\therefore PV = \frac{60\,000}{1.01^{20}}$$
$$= \$49\,172.66\dots$$
$$\Rightarrow C$$

14. **A** Simple Interest (137268)

$$\begin{aligned}\text{Interest} &= 800 \times \frac{x}{12} \times \frac{3}{100} \\ &= 2x\end{aligned}$$

$\Rightarrow A$

15. **A** Earning and Spending Money (002647)

Rent assistance = \$119.40

Youth allowance = 402.70 less 50c per dollar (income > \$236)

$$= 402.70 - 0.5(300 - 236)$$

$$= 402.70 - 32$$

$$= \$370.70$$

$\therefore$ Total Payment = 119.40 + 370.70

$$= \$490.10$$

$\Rightarrow D$

Mean mark 20%!

16. **A** Cartesian Plane (010503)

$$2x - 3y = 8$$

$$3y = 2x - 8$$

$$y = \frac{2}{3}x - \frac{8}{3}$$

$$m = \frac{2}{3}$$

$$\therefore m_{\text{perp}} = -\frac{3}{2} \quad (m_1 m_2 = -1 \text{ for } \perp \text{ lines})$$

Equation of line $m = -\frac{3}{2}$ through $(2, 0)$:

$$y - y_1 = m(x - x_1)$$

$$y - 0 = -\frac{3}{2}(x - 2)$$

$$y = -\frac{3}{2}x + 3$$

$$2y = -3x + 6$$

$$3x + 2y = 6$$

$\Rightarrow B$

17. **D** Right-Angled Trig (232468)

Let x = unknown side

By Pythagoras:

$$2^2 = x^2 + 1^2$$

$$x^2 = 4 - 1$$

$$x = \sqrt{3} \quad (x > 0)$$

$$\cos 30^\circ = \frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{3}}{2}$$

18. **C** Right-Angled Trig (232647)

$$\tan 70^\circ = \frac{x}{10.2}$$

$$x = 10.2 \times \tan 70^\circ$$

$$= 28.0 \quad (\text{to 1 d.p.})$$

19. C Right-Angled Trig (232618)

i. By Pythagoras,

$$\begin{aligned}x^2 + 8^2 &= 10^2 \\x^2 &= 100 - 36 \\&= 64 \\x &= 8\end{aligned}$$

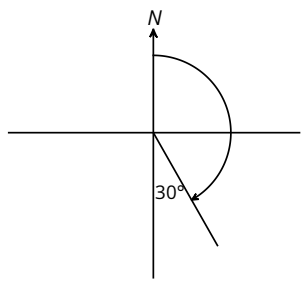
$$\therefore \cos\theta = \frac{8}{10} = \frac{4}{5}$$

ii. $\theta = \cos^{-1}\left(\frac{4}{5}\right)$
 $= 36.869\dots^\circ$
 $= 36.87^\circ$ (to 2 d.p.)

20. C Cartesian Plane (228340)

$$\begin{aligned}C(-3, -5), \quad D(-5, 1) \\M &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \\&= \left(\frac{-3 - 5}{2}, \frac{-5 + 1}{2}\right) \\&= (-4, -2)\end{aligned}$$

21. C Right-Angled Trig (245025)



$$\begin{aligned}\text{S}30^\circ\text{E} &= 90 + 60 \\&= 150^\circ\end{aligned}$$

22. C Cartesian Plane (228485)

Line goes through $(3, -2)$ and $(-1, 4)$.

Using the gradient formula:

$$\begin{aligned}m &= \frac{y_2 - y_1}{x_2 - x_1} \\&= \frac{-2 - 4}{3 - (-1)} \\&= -\frac{3}{2}\end{aligned}$$

Find equation through $(3, -2)$, $m = -\frac{3}{2}$:

$$\begin{aligned}y - y_1 &= m(x - x_1) \\y - (-2) &= -\frac{3}{2}(x - 3) \\2(y + 2) &= -3(x - 3) \\2y + 4 &= -3x + 9 \\2y + 3x - 5 &= 0\end{aligned}$$

23. C Right-Angled Trig (157388)

a. $\tan\theta = \frac{8}{10}$
 $\theta = \tan^{-1}\frac{8}{10}$
 $= 38.659\dots$
 $= 39^\circ$ (nearest degree)

b. Using Pythagoras:

$$\begin{aligned}x &= \sqrt{8^2 + 10^2} \\&= 12.806\dots \\&= 12.8 \text{ (to 1 d.p.)}\end{aligned}$$

24. **C** Compound Interest (225910)

Using $FV = PV(1 + r)^n$

$$r = 3\%, n = 2$$

$$20\,000 = PV(1 + 0.03)^2$$

$$\begin{aligned}\therefore PV &= \frac{20\,000}{1.03^2} \\ &= \$18\,851.92\end{aligned}$$

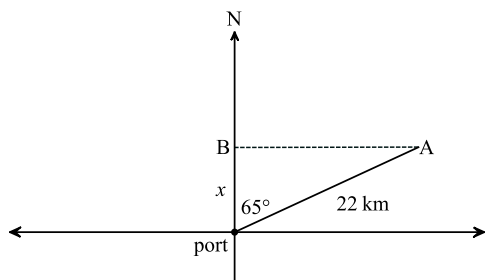
25. **C** Cartesian Plane (228425)

$$(6, -5), (0, 3)$$

Using the distance formula:

$$\begin{aligned}d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(6 - 0)^2 + (-5 - 3)^2} \\ &= \sqrt{36 + 64} \\ &= \sqrt{100} \\ &= 10 \text{ units}\end{aligned}$$

26. **B** Right-Angled Trig (245577)



Let x = distance from port to Boat B

$$\begin{aligned}\cos 65^\circ &= \frac{x}{22} \\ x &= 22 \times \cos 65^\circ \\ &= 9.2976\dots \\ &= 9.30 \text{ km}\end{aligned}$$

27. **B** Simple Interest (253309)

$$\text{Total interest paid} = 4400 - 4000 = \$400$$

$$I = Prn$$

$$400 = 4000 \times r \times 2$$

$$r = \frac{400}{8000}$$

$$= 0.05$$

$$= 5.0\%$$

28. **B** Compound Interest (254448)

a. Interest rate (r) = $\frac{8.2\%}{2} = 4.1\% = 0.041$ (per 6 months)

$$\text{Compounding periods } (n) = 2 \times 2 = 4$$

$$\begin{aligned}FV &= PV(1 + r)^n \\ &= 3000(1.041)^4 \\ &= 3523.093\dots \\ &= \$3523.09 \text{ (nearest cent)}\end{aligned}$$

b. Compounding periods (n) = $4 \times 2 = 8$

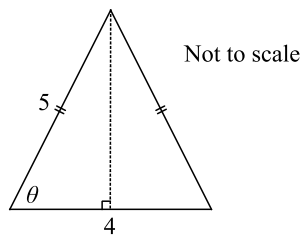
$$\begin{aligned}FV &= 3000(1.041)^8 \\ &= 4137.396\dots\end{aligned}$$

$$\begin{aligned}\therefore \text{Interest earned} &= 4137.40 - 3000 \\ &= \$1137.40 \text{ (nearest cent)}\end{aligned}$$

29. **B+** Depreciation (225971)

$$\begin{aligned}S &= V_0(1 - r)^n \\ &= 76\,500 \left(1 - \frac{14}{100}\right)^6 \\ &= 76\,500(0.86)^6 \\ &= \$30\,949.39 \\ &= \$30\,949 \text{ (nearest dollar)}\end{aligned}$$

30. **B+** Right-Angled Trig (232622)



$$\cos \theta = \frac{2}{5}$$

$$\theta = \cos^{-1} \left(\frac{2}{5} \right)$$

$$= 66.421 \dots^\circ$$

$$= 66^\circ 25' \text{ (to 1 d.p.)}$$

31. **A** Cartesian Plane (228433)

$$(p, 3), (1, p)$$

Using the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$10 = \sqrt{(p - 1)^2 + (3 - p)^2}$$

$$10 = \sqrt{p^2 - 2p + 1 + 9 - 6p + p^2}$$

$$10 = \sqrt{2p^2 - 8p + 10}$$

$$100 = 2p^2 - 8p + 10$$

$$0 = 2p^2 - 8p - 90$$

$$0 = p^2 - 4p - 45$$

$$0 = (p - 9)(p + 5)$$

$$\therefore p = 9 \text{ or } -5$$

32. **A** Compound Interest (177885)

a. $r = \frac{6\%}{12} = 0.5\% = 0.005 \text{ per month}$

$$n = 12$$

$$\begin{aligned} FV &= PV(1 + r)^n \\ &= 35\,000(1 + 0.005)^{12} \\ &= \$37\,158.72 \end{aligned}$$

b. $I = Prn$

$$2158.72 = 35\,000 \times r \times 1$$

$$r = \frac{2158.72}{35\,000}$$

$$= 0.06167 \dots$$

$$= 6.17\% \text{ (to 2 d.p.)}$$

Mean mark part (b) 36%.

33. **A+** Simple Interest (225668)

$$\begin{aligned} \text{Interest earned} &= 10\,350 - 9000 \\ &= \$1350 \end{aligned}$$

Using $I = Prn$,

$$1350 = 9000 \times r \times 4$$

$$\therefore r = \frac{1350}{4 \times 9000}$$

$$= 0.0375$$

$$= 3.75\%$$