MATHEMATICS
Stage 5

Measurement and Space, Properties of Geometrical Figures
Similarity
Special Properties

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Exam Equivalent Time: 139.5 minutes *(based on allocation ~1.25 min/mark)*

Worked Solutions

Worked Solutions

1. D Similarity (015093)

Take two corresponding sides

In ΔA: 3 cm

In ΔB: $1\frac{1}{2}$ cm

$\therefore$ Scale factor converting ΔA to $\Delta B = \dfrac{1}{2}$

$\Rightarrow A$

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2. D Special Properties (249437)

An equilateral triangle has all angles $= 60^\circ$.

$\therefore$ A right-angled, equilateral triangle is impossible.

$\Rightarrow B$

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3. D Similarity (248489)

$2:40 \quad \Rightarrow \quad 1:20$

1 cm represents 20 cm

$\Rightarrow C$

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4. C Special Properties (249611)

By elimination:

A and B clearly incorrect.

C true if all sides are equal (rhombus) but not true for all parallelograms.

Line PS must be parallel to line QR.

$\Rightarrow D$

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5. C Special Properties (249440)

A triangle's angles add up to 180°, and a reflex angle is greater than 180°.

$\therefore$ The third angle cannot be reflex.

$\Rightarrow D$

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6. C Similarity (248499)

$$\begin{aligned} 12 \text{ cm} &: 24 \text{ mm} \\ 120 \text{ mm} &: 24 \text{ mm} \\ 5 &: 1 \\ 1 \text{ cm} &: 2 \text{ mm} \quad \text{(possible options)} \end{aligned}$$

$\Rightarrow B$

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7. C Special Properties (249600)

Kite.
(Note that a rectangle has a pair of equal opposite sides)

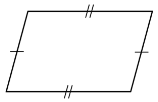
$\Rightarrow C$

8. C Special Properties (249616)

Consider each option:

A: rhombus diagonals are perpendicular but not always equal

B: parallelogram diagonals not always equal (see below)



C: always true (see above)

D: at least 1 pair of opposite sides of a trapezium are not equal

$\Rightarrow C$

9. C Similarity (197628)

$$\text{Scale factor } = \frac{3}{2} = 1.5$$

$$\therefore\ x = 1.5 \times 12 = 18$$

Alternate solution
Using sides of similar figures in the same ratio:

$$\frac{x}{12} = \frac{3}{2}$$

$$x = 12 \times \left(\frac{3}{2}\right)$$

$$x = 18$$

$\Rightarrow C$

10. C Similarity (248529)

$$\begin{aligned} 6 \text{ cm} &= 3 \text{ m} \\ &= 300 \text{ cm} \\ \therefore 1 \text{ cm} &= 300 \div 6 \\ &= 50 \text{ cm} \end{aligned}$$

$\Rightarrow D$

11. B Similarity (002012)

Mean mark 38%

We know $\Delta ABC \,|||\, \Delta DEF$

$$\therefore\ \frac{AB}{AC} = \frac{y}{10} = \frac{DE}{DF} = \frac{x}{15}$$

$$\frac{x}{15} = \frac{y}{10}$$

$$x = y \times \frac{15}{10}$$

$\Rightarrow C$

12. B Special Properties (249748)

Consider each option:

A: Isosceles (not scalene) have two equal angles.

B: Only opposite angles in a parallelogram are equal.

C: At least one pair of opposite sides of a trapezium are not equal.

D: Rhombuses have perpendicular diagonals.

$\Rightarrow D$

13. B Special Properties (249577)

$AB \neq AC \neq BC$

$\angle BCA = 90^\circ$

ΔABC is both right-angled and scalene

$\therefore$ Right-angled is the BEST description

$\Rightarrow D$

14. B Special Properties (249815)

$3x = 180^\circ \ \Rightarrow\ x = 60^\circ$

$AO = OC = OD = OB$ (radii of circle)

$\Rightarrow$ Since angles opposite equal sides of a triangle are equal, all triangle angles can be found to equal 60°.

$\therefore$ The three triangles are equilateral.

$\Rightarrow D$

15. B+ Similarity (014288)

Triangles are similar (equiangular)

In smaller triangle:

$$h^2 = 2^2 + 3^2$$
$$= 13$$
$$h = \sqrt{13}$$

$$\frac{x}{\sqrt{13}} = \frac{8}{2} \quad \text{(sides of similar } \Delta\text{s in same ratio)}$$
$$x = \frac{8\sqrt{13}}{2}$$
$$= 14.422\ldots$$

$\Rightarrow D$

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16. B+ Similarity (060339)

Triangles are similar (equiangular)

Using similar ratios:

$$\frac{W}{7.1} = \frac{20.3}{8.1}$$
$$\therefore W = \frac{20.3 \times 7.1}{8.1}$$
$$= 17.79\ldots$$

$\Rightarrow A$

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17. D Special Properties (249606)

Since the quadrilateral was divided into two triangles

$\Rightarrow$ Sum of internal angles $= 2 \times 180 = 360^\circ$

$$\therefore x = 360 - (103 + 88 + 62)$$
$$= 360 - 253$$
$$= 107^\circ$$

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18. C Similarity (248513)

$$\text{Scale factor} = \frac{\text{new width}}{\text{old width}}$$
$$= \frac{50}{25}$$
$$= 2$$

$$\therefore \text{ New height} = 2 \times 10$$
$$= 20 \text{ cm}$$

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19. C Special Properties (249848)

Sum of exterior angles $= 360^\circ$

$$x = 360 - (65 + 85 + 80 + 70)$$
$$= 360 - 300$$
$$= 60^\circ$$

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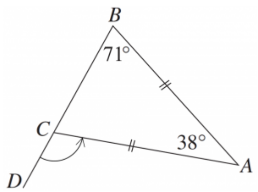
20. C Special Properties (029155)



$$\angle ABC = \frac{1}{2}(180 - 38) \quad \text{(base angle of isosceles } \Delta ABC)$$
$$= 71^\circ$$

$$\therefore \angle ACD = 71 + 38 \quad \text{(exterior angle of } \Delta ABC)$$
$$= 109^\circ$$

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21. C Similarity (248244)

Similar triangles are equiangular.

Calculate triangle angles of each option.

Triangle A: 55, 55, 70

Triangle B: 50, 65, 65

Triangle C: 50, 55, 65

Triangle D: 55, 55, 70

$\therefore$ Triangles A and D are similar.

22. C Similarity (178755)

Mean mark 32%.

Dimension of larger shape:

Width $= 16 \times 1.5 = 24$ cm

Height $= 9 \times 1.5 = 13.5$ cm

Triangle height $= 2.5 \times 1.5 = 3.75$ cm

$$\therefore \text{ Area} = 24 \times (13.5 - 3.75) + \frac{1}{2} \times 24 \times 3.75$$

$$= 279 \text{ cm}^2$$

23. C Special Properties (249920)

Sum of exterior angles $= 360^\circ$

Since the hexagon is regular, all external angles are equal.

$$\therefore x^\circ = \frac{360}{6} = 60^\circ$$

24. C Similarity (249036)

$$\frac{RP}{AC} = \frac{18}{12} = \frac{3}{2}$$

$$\frac{QR}{BA} = \frac{12}{8} = \frac{3}{2}$$

$$\frac{PQ}{CB} = \frac{9}{6} = \frac{3}{2}$$

$\therefore \Delta PQR\ |||\ \Delta CBA$ (three pairs of sides in proportion)

25. C Special Properties (249868)

Sum of exterior angles $= 360^\circ$

Since the pentagon is regular, all external angles are equal.

$$\therefore x^\circ = \frac{360}{5} = 72^\circ$$

26. C Similarity (249073)

$$\frac{AB}{GH} = \frac{11}{22} = \frac{1}{2}$$

$$\frac{BC}{FG} = \frac{3.5}{7} = \frac{1}{2}$$

$\angle ABC = \angle FGH = 95^\circ$ (given)

$\therefore \Delta ABC\ |||\ \Delta HGF$ (sides adjacent to equal angles in proportion)

27. C Similarity (248235)

Since trapeziums are similar:

$$\frac{EF}{BC} = \frac{AG}{AD}$$

$$\frac{EF}{8} = \frac{3}{12}$$

$$EF = \frac{3 \times 8}{12}$$

$$= 2 \text{ m}$$

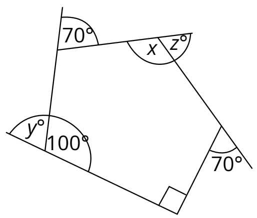
28. C Similarity (249054)

$\angle ABE = \angle CBD$ (vertically opposite)

$\angle AEB = \angle BCD$ (alternate, $AE \parallel CD$)

$\therefore \Delta ABE\ |||\ \Delta DBC$ (equiangular)

29. B Special Properties (249947)



NOT TO SCALE

$y^\circ = 180 - 100 = 80^\circ$ (180° in straight line)

Sum of exterior angles $= 360^\circ$

$$z^\circ = 360 - (70 + 70 + 90 + 80)$$

$$= 360 - 310$$

$$= 50^\circ$$

$\therefore x^\circ = 180 - 50 = 130^\circ$ (180° in straight line)

30. B Similarity (248523)

Ratio is $1:400$

Converting to centimetres:

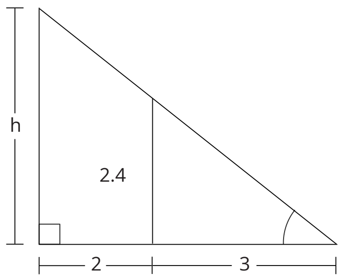
$120 \text{ m} = 120 \times 100 = 12\ 000 \text{ cm}$

$\therefore$ Length of the scale diagram

$= 12\ 000 \div 400$

$= 30 \text{ cm}$

31. B Similarity (248223)



Triangles share common angle and both have right angles

$\Rightarrow$ Triangles are similar (equiangular)

Since corresponding sides are in similar ratios:

$$\frac{h}{5} = \frac{2.4}{3}$$

$$\therefore h = \frac{5 \times 2.4}{3}$$

$= 4$ metres

32. B Special Properties (249785)

Consider regular pentagon:

Sum of internal angles (formula)

$= (n-2) \times 180$

$= 3 \times 180$

$= 540^\circ$

$$\therefore x = \frac{540}{5}$$

$= 108^\circ$

TIP: Two quadrilaterals joining ***does not*** make the internal angle sum = 2 × 360°!

33. B Similarity (249089)

Find unknown side (x) of smaller triangle

Using Pythagoras:

$x = \sqrt{5^2 - 4^2} = \sqrt{9} = 3$

$$\frac{AC}{DE} = \frac{5}{15} = \frac{1}{3}$$

$$\frac{BC}{EF} = \frac{3}{9} = \frac{1}{3}$$

$\angle ABC = \angle DEF = 90^\circ$ (given)

$\therefore \Delta ABC \ ||| \ \Delta DEF$ (hypotenuse and second side of right-angled triangle in proportion)

34. B Similarity (248787)

In Triangle I, using Pythagoras:

$\text{Base} = \sqrt{5^2 - 3^2}$

$= 4$

Triangle I ||| Triangle II (given)

$\Rightarrow$ corresponding sides are in the same ratio

Mean mark 29%.

$\text{Scale factor} = \dfrac{6}{2} = 2$

$\text{Scale factor (Area)} = 2^2 = 4$

$\therefore$ Area (Triangle II) $= 4 \times$ Area of triangle I

$= 4 \times \dfrac{1}{2} \times 3 \times 4$

$= 24 \text{ cm}^2$

35. B Similarity (029440)

Both triangles have right angles and a common angle to the ground.

$\therefore$ Triangles are similar (equiangular)

Let $x =$ length of tower shadow

$\dfrac{x}{5} = \dfrac{19.2}{1.65}$ (corresponding sides of similar triangles)

$x = \dfrac{5 \times 19.2}{1.65}$

$= 58.1818\ldots$

$= 58 \text{ m}$ (nearest m)

36. B Similarity (249100)

$\angle ACD = \angle BEA$ (given)

$\angle BAE$ is common to ΔACD and ΔBEA

$\therefore \Delta ACD ||| \Delta BEA$ (equiangular)

37. B Similarity (249231)

$\angle BDC = 180 - (88 + 44) = 48^\circ$ (angle sum of Δ)

$\angle CAD = \angle CDB = 48^\circ$

$\angle ACD = 44^\circ$ is common to ΔACD and ΔBCD

$\therefore \Delta ACD ||| \Delta DCB$ (equiangular)

38. B+ Similarity (249334)

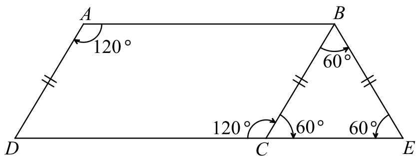
i. Scale factor $= \dfrac{\text{Diameter B}}{\text{Diameter A}} = \dfrac{0.8}{2.4} = \dfrac{1}{3}$

ii. Scale factor (B to A) $= \dfrac{\text{Diameter A}}{\text{Diameter B}} = \dfrac{2.4}{0.8} = 3$

Scale factor (Area) $= 3^2 = 9$

$\therefore$ Area of Circle A $= 9 \times$ Area of Circle B

39. B+ Special Properties (027720)



$BC = AD$ (opposite sides of parallelogram $ABCD$)

$\angle BCD = 120^\circ$ (opposite angles of parallelogram $ABCD$)

$\angle BCE = 60^\circ$ ($\angle DCE$ is a straight angle)

$\angle CEB = 60^\circ$ (base angles of isosceles ΔBCE)

$\angle CBE = 60^\circ$ (angle sum of ΔBCE)

$\therefore \Delta BCE$ is equilateral

40. B+ Similarity (249243)

i. Dimensions increase by a factor of 2

$\Rightarrow$ Volume increases by a factor of $2^3 = 8$

ii. Dimensions decrease by two-thirds

$\Rightarrow$ i.e. adjust dimensions by a factor of $\dfrac{1}{3}$

$\Rightarrow$ Volume decreases by a factor of $\left(\dfrac{1}{3}\right)^3 = \dfrac{1}{27}$

41. B+ Special Properties (249756)

ΔADC and ΔABC are equiangular triangles.

$\angle DCA = \angle ACB = 60^\circ$

$\therefore \angle DCB = 60 \times 2$

$= 120^\circ$

42. A Similarity (003173)

Mean mark 24%

Both triangles have right-angles with a common (ground) angle.

$\therefore$ Triangles are similar (equiangular)

Since corresponding sides are in the same ratio

$$\frac{d}{1.5} = \frac{d+3}{4}$$

$$4d = 1.5(d+3)$$

$$8d = 3(d+3)$$

$$= 3d + 9$$

$$5d = 9$$

$$\therefore d = \frac{9}{5}$$

$$= 1.8 \text{ m}$$

43. A Special Properties (249809)

Sum of internal angles of pentagon

$$\begin{aligned} &= (n-2) \times 180 \\ &= 3 \times 180 \\ &= 540^\circ \end{aligned}$$

Internal angle in pentagon

$$\begin{aligned} &= \frac{540}{5} \\ &= 108^\circ \end{aligned}$$

$$\begin{aligned} \therefore x &= 360 - (108 + 90 + 60) \\ &= 102^\circ \end{aligned}$$

44. A Similarity (159063)

Consider ΔABC:

$$\begin{aligned} \text{Area} &= \frac{1}{2} \times AB \times BC \\ 20 &= \frac{1}{2} \times 8 \times BC \\ \therefore BC &= 5 \end{aligned}$$

Using Pythagoras in ΔABC:

$$AC = \sqrt{8^2 + 5^2} = \sqrt{89}$$

Mean mark 11%.

Since $\Delta ABC \mid\mid\mid \Delta DEF$,

$$\begin{aligned} \frac{AC}{BC} &= \frac{DF}{EF} \\ \frac{\sqrt{89}}{5} &= \frac{DF}{4} \\ \therefore DF &= \frac{4\sqrt{89}}{5} \\ &= 7.547\ldots \\ &= 7.55 \text{ cm (to 2 d.p.)} \end{aligned}$$

45. A Similarity (248267)

Circles are similar

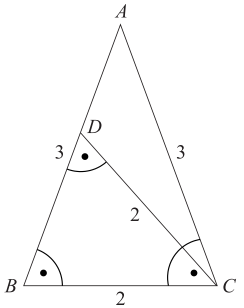
$$\begin{aligned} \text{Arc } ST &= \frac{2}{3} \times \text{arc } CD \\ &= \frac{2}{3} \times 21 \\ &= 14 \text{ cm} \end{aligned}$$

$$\begin{aligned} \angle SOR &= 180 - 45 \\ &= 135^\circ \\ &= 3 \times \angle SOT \end{aligned}$$

$$\begin{aligned} \therefore \text{Arc } RS &= 3 \times \text{arc } ST \\ &= 3 \times 14 \\ &= 42 \text{ cm} \end{aligned}$$

46. A Similarity (118846)

i. Prove $\Delta ABC \ ||| \ \Delta CBD$



NOT TO SCALE

ΔABC is isosceles:

$\angle ABC = \angle ACB$ (angles opposite equal sides)

ΔCBD is isosceles:

$\angle CBD = \angle CDB$ (angles opposite equal sides)

Since $\angle ABC = \angle CBD$

$\therefore \Delta ABC \ ||| \ \Delta CDB$ (equiangular)

ii. Using ratios of similar triangles

$$\frac{DB}{CB} = \frac{BC}{AC}$$

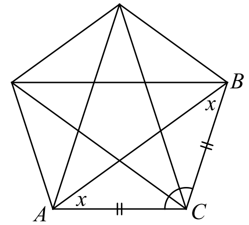
$$\frac{(3-AD)}{2} = \frac{2}{3}$$

$$3 - AD = \frac{4}{3}$$

$$\therefore AD = \frac{5}{3}$$

47. A Special Properties (249772)

Consider the triangle ABC in the pentagon:



STRATEGY: The internal angle sum is $3 \times 180 = 540$ (since 3 triangles can be drawn internally from one point).

Total degrees in a pentagon

$= 3 \times 180$

$= 540^\circ$

Internal angle $= \dfrac{540}{5} = 108^\circ$ (regular pentagon)

ΔABC is isosceles

$\therefore x + x + 108 = 180$

$2x = 72$

$x = 36^\circ$