



NORMANHURST BOYS HIGH SCHOOL

STAGE 5.1-5.3 MATHEMATICS

2020 Year 9 Yearly Examination

Wednesday October 28, 2020

General instructions

- Working time – 80 minutes.
- Write your answers in the space provided.
- Marks may be deducted for careless or poorly arranged work.
- Write using blue or black pen. Where diagrams are to be sketched, these may be done in pencil.
- NESA approved calculators may be used.
- All necessary working should be shown in every question.
- Attempt **all** questions.

Class (please ✓)

- ☐ 9MAT.N – Miss Lee
- ☐ 9MAT.B – Mr Ho/Miss Ham
- ☐ 9MAT.H – Mr Lam
- ☐ 9MAT.S – Mr Sun

NAME:

Marker's use only.

QUESTION	1	2	3	4	5	6	7	8	9	Total	%
MARKS	$\frac{1}{11}$	$\frac{1}{14}$	$\frac{1}{16}$	$\frac{1}{9}$	$\frac{1}{8}$	$\frac{1}{14}$	$\frac{1}{7}$	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{92}$	

Question 1 (11 Marks)**Marks**

(a) Factorise fully:

i. $x^3 - x$.

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ii. $x^2 + x - 42$

2

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iii. $3x^2 - 20x + 12$

2

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(b) Simplify fully:

i. $\frac{3x-7}{5} - \frac{x+6}{4}$.

2

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ii. $\frac{y^2 - 5y + 6}{(y-3)^2} \times \frac{y}{y^2 - 2y}$.

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Question 2 (14 Marks)**Marks**

- (a) Rewrite $\frac{1}{\sqrt[4]{y}}$ in index form with positive indices only. **1**

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- (b) Fully simplify each of the following expressions, expressing your answer without negative indices.

i. $t^2 \times \sqrt[3]{t}$ **2**

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ii. $2p^{-7} \div 8p^{-4} \times (3p)^0$ **2**

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(c) Fully simplify: $2\sqrt{75} + 3\sqrt{3}$ **2**

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(d) Expand and fully simplify: $(\sqrt{2} + 3)^2$ **2**

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- (e) Express the following with a rational denominator, giving your answer in simplest form:

i. $\frac{10}{\sqrt{5}}$ **2**

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ii. $\frac{2}{\sqrt{7} - \sqrt{3}}$ **3**

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Examination continues overleaf...

Question 3 (16 Marks)**Marks**(a) Solve for x :

i. $\frac{x-2}{x+2} = 5.$

2

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ii. $4(x+6) \geq 3(3x-10).$

2

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iii. $8^{5x+6} = 4^{3x}.$

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(b) Make p the subject of the expression:**2**

$$r = p^2 + k$$

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Examination continues overleaf...

(c) Solve for x :

i. $x^2 - 11x + 10 = 0$

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ii. $(x - 3)^2 = 16$.

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iii. $x^2 + 6x - 14 = 0$ (by completing the square)

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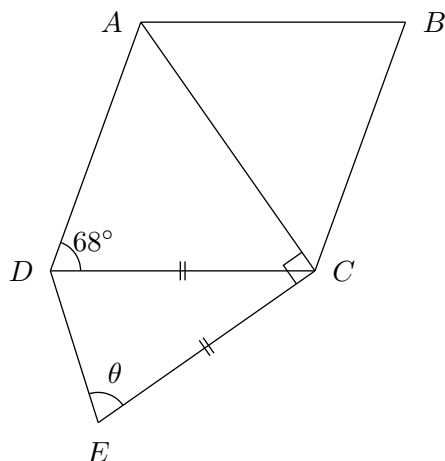
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Examination continues overleaf...

Question 4 (9 Marks)**Marks**

- (a) $ABCD$ is a rhombus. $DC = EC$. $\angle ADC = 68^\circ$ and $\angle ACE = 90^\circ$.

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Find the value of θ , giving full geometrical reasoning.

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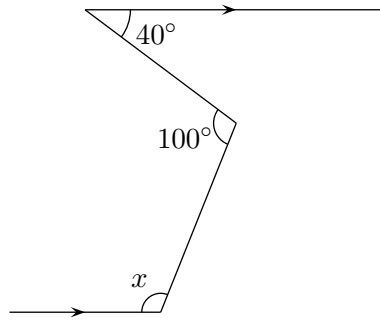
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Examination continues overleaf...

- (b) Find the value of x in the diagram, giving full geometrical reasoning.

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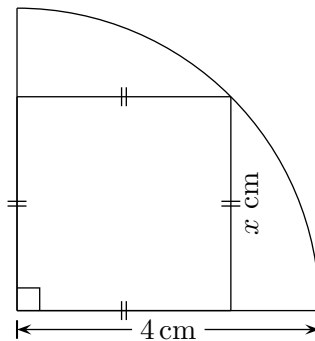
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- (c) A square of side length x is drawn in a quadrant of a circle with radius 4 cm.

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Find the value of x . Geometrical reasoning is not required.

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Examination continues overleaf...

Question 5 (8 Marks)**Marks**

- (a) Find the gradient and the
- y
- intercept of the line

2

$$2x - 3y + 12 = 0$$

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- (b)
- A
- is the point
- $(-1, -3)$
- and
- B
- is the point
- $(3, 2)$
- .

- i. Find the equation of
- AB
- , giving your answer in general form.

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- ii. What is the gradient of any line perpendicular to
- AB
- ?

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- iii. Find the equation of the line perpendicular to
- AB
- which passes through
- B
- .
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- Give your answer in the form
- $y = mx + c$
- .

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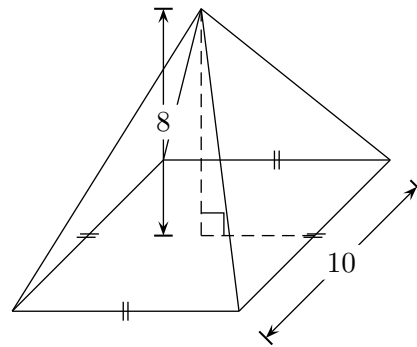
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Question 6 (14 Marks)**Marks**

- (a) For the following square pyramid, find the:



- i. volume

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- ii. exact surface area

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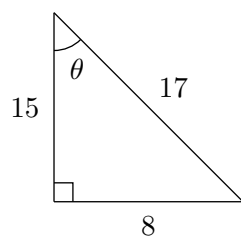
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- (b) State the value of
- $\sin \theta$
- in the diagram below.

1

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Examination continues overleaf...

- (c) Town A is 2.6 km east of Town C , Town C is 1.7 km due south of the Town B . 2

By drawing a diagram or otherwise, find the bearing of Town B from Town A .

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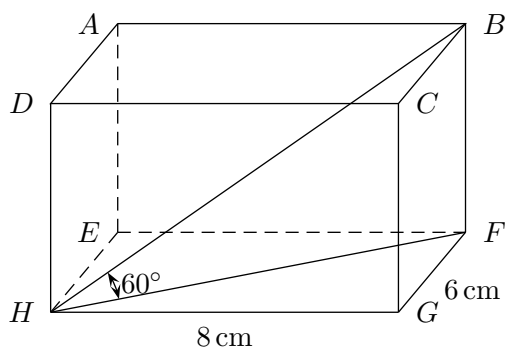
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- (d) In the rectangular prism shown, $FG = 6$ cm, $HG = 8$ cm and $\angle BHF = 60^\circ$.



- i. Find the size of $\angle FHG$, correct to the nearest degree. 2

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- ii. Find the length of FH . 1

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- iii. Hence, find the length of BH . 2

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- iv. Find the *exact* length of BF . 2

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Examination continues overleaf...

Question 7 (7 Marks)**Marks**

- (a) Solve the following pair of simultaneous equation by using an appropriate method: **3**

$$\begin{cases} 2x - 3y = -8 \\ 5x + 4y = 3 \end{cases}$$

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- (b) A mathematics test had 20 questions. Some questions are worth 2 marks, whilst others were worth 3 marks. The test had a total of 52 marks. **4**

By establishing a pair of simultaneous equations and solving them, find the number of each type of question in the test.

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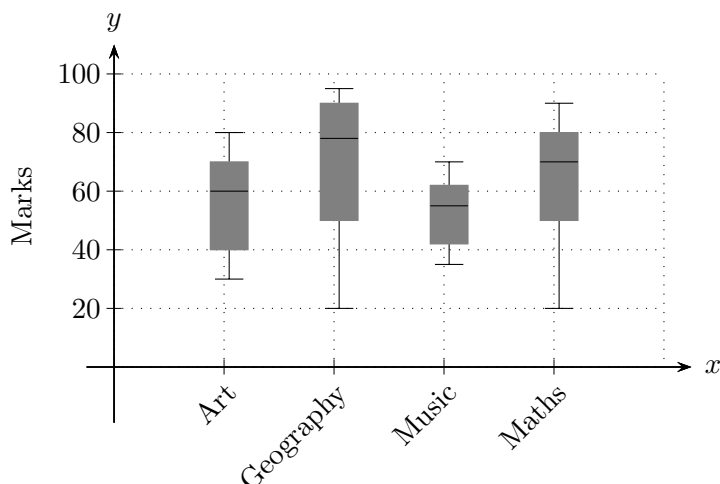
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Question 8 (6 Marks)**Marks**

- (a) The following box and whisker plots show examination results in four subjects.



- i. Which subject had the largest range? 1
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 - ii. Which subject has the highest median? 1
.....
 - iii. Which subject has an upper quartile of 80? 1
.....
 - iv. Which subject has an interquartile range of 20? 1
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- (b) Find the amount of interest earned after 3 years, if Mr Ho deposits \$1 500 into an account earning 1.5% per annum, compounded daily. 2

You may assume that this scenario contains no leap years for the purposes of the calculation.

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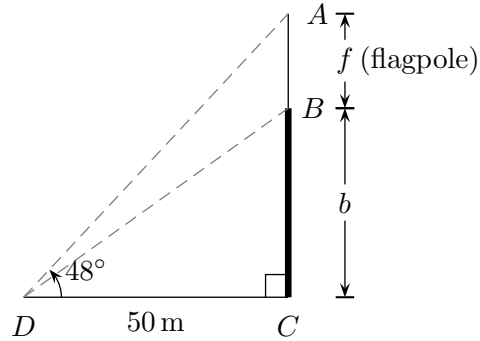
Question 9 (7 Marks)**Marks**

- (a) A large flagpole
- AB
- stands on top of a building.

3

From D , located 50 metres from C , the angle of elevation to the top of the building is 45° . The angle of elevation from D to the top of the flagpole is 48° .

Let b be the height of the building, and f be the length of the flagpole.



Find the height of the flagpole, correct to the nearest metre.

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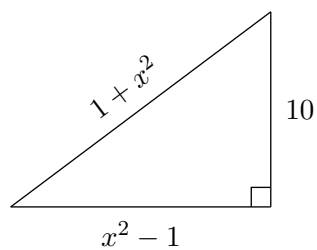
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Examination continues overleaf...

- (b) Find the value(s) of x . *Hint:* there may be some algebraic expansions involved.

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End of paper.

Additional writing page

Suggested Solutions

Question 1

- (a) i. (2 marks)
 ✓ [1] for taking out the common factor.
 ✓ [1] for final answer.

$$\begin{aligned}x^3 - x &= x(x^2 - 1) \\ &= x(x - 1)(x + 1)\end{aligned}$$

- ii. (2 marks)
 ✓ [-1] for each newly introduced error.

$$x^2 + x - 42 = (x - 6)(x + 7)$$

- iii. (2 marks)
 ✓ [-1] for each newly introduced error.

$$3x^2 - 20x + 12 = (3x - 2)(x - 6)$$

- (b) i. (2 marks)
 ✓ [1] for rewriting the fraction in unsimplified form with a new denominator.
 ✓ [1] for final answer.

$$\begin{aligned}&\frac{3x - 7}{5} - \frac{x + 6}{4} \\ &= \frac{4(3x - 7) - 5(x + 6)}{20} \\ &= \frac{12x - 28 - 5x - 30}{20} \\ &= \frac{7x - 58}{20}\end{aligned}$$

- ii. (3 marks)
 ✓ [1] for each correct factorisation of the required expressions.
 ✓ [1] for final answer.

$$\begin{aligned}&\frac{y^2 - 5y + 6}{(y - 3)^2} \times \frac{y}{y^2 - 2y} \\ &= \frac{\cancel{(y - 3)}\cancel{(y - 2)}}{(y - 3)^2} \times \frac{\cancel{y}}{\cancel{y}(y - 2)} \\ &= \frac{1}{y - 3}\end{aligned}$$

Question 2

- (a) (1 mark)

$$\frac{1}{\sqrt[4]{y}} = \frac{1}{y^{\frac{1}{4}}}$$

- (b) i. (2 marks)
 ✓ [1] for correct conversion of $\sqrt[3]{t}$ into index form.
 ✓ [1] for final answer.

$$\begin{aligned}t^2 \times \sqrt[3]{t} &= t^2 \times t^{\frac{1}{3}} \\ &= t^{\frac{7}{3}}\end{aligned}$$

- ii. (2 marks)
 ✓ [-1] for each newly introduced error.

$$\begin{aligned}2p^{-7} \div 8p^{-4} \times (3p)^0 &= \frac{1}{4}p^{-3} \times 1 \\ &= \frac{1}{4p^3}\end{aligned}$$

- (c) (2 marks)
 ✓ [1] for simplifying $2\sqrt{75}$.
 ✓ [1] for final answer.

$$\begin{aligned}2\sqrt{75} + 3\sqrt{3} &= 2 \times 5\sqrt{3} + 3\sqrt{3} \\ &= 13\sqrt{3}\end{aligned}$$

- (d) (2 marks)
 ✓ [-1] for each newly introduced error.

$$\begin{aligned}(\sqrt{2} + 3)^2 \\ &= 2 + 6\sqrt{2} + 9 \\ &= 11 + 6\sqrt{2}\end{aligned}$$

- (e) i. (2 marks)
 ✓ [1] for multiplying numerator and denominator by $\sqrt{5}$.
 ✓ [1] for final answer.

$$\frac{10}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{10\sqrt{5}}{5} = 2\sqrt{5}$$

ii. (3 marks)

- ✓ [1] for multiplying numerator and denominator by $\sqrt{7} + \sqrt{3}$.
- ✓ [1] for further simplification.
- ✓ [1] for fully simplest form.

$$\begin{aligned} & \frac{2}{\sqrt{7} - \sqrt{3}} \times \frac{(\sqrt{7} + \sqrt{3})}{(\sqrt{7} + \sqrt{3})} \\ &= \frac{2(\sqrt{7} + \sqrt{3})}{7 - 3} \\ &= \frac{\sqrt{7} + \sqrt{3}}{2} \end{aligned}$$

Question 3

(a) i. (2 marks)

- ✓ [1] for multiplying $(x + 2)$ across.
- ✓ [1] for final answer.

$$\begin{aligned} \frac{x - 2}{x + 2} &= 5 \\ x - 2 &= 5(x + 2) \\ x - 2 &= 5x + 10 \\ -x & \quad -x \\ 4x &= -12 \\ x &= -3 \end{aligned}$$

ii. (2 marks)

- ✓ [1] for correct expansion on both sides.
- ✓ [1] for final answer.

$$\begin{aligned} 4(x + 6) &\geq 3(3x - 10) \\ 4x + 24 &\geq 9x - 30 \\ -4x & \quad -4x \\ 5x - 30 &\leq 24 \\ +30 & \quad +30 \\ 5x &\leq 54 \\ \div 5 & \quad \div 5 \\ x &\leq \frac{54}{5} \end{aligned}$$

iii. (3 marks)

- ✓ [1] for changing 8^{5x+6} into a power of 2.
- ✓ [1] for changing 4^{3x} into a power of 2.

✓ [1] for final answer.

$$\begin{aligned} 8^{5x+6} &= 4^{3x} \\ (2^3)^{5x+6} &= (2^2)^{3x} \\ 2^{3(5x+6)} &= 2^{6x} \\ 3(5x + 6) &= 6x \\ 15x + 18 &= 6x \\ 9x &= -18 \\ x &= -2 \end{aligned}$$

(b) (2 marks)

- ✓ [1] for $\sqrt{r - k}$.
- ✓ [1] for $\pm$ symbol.

$$\begin{aligned} r &= p^2 + k \\ p^2 &= r - k \\ p &= \pm\sqrt{r - k} \end{aligned}$$

(c) i. (2 marks)

- ✓ [1] for correct factorisation.
- ✓ [1] for both correct solutions.

$$\begin{aligned} x^2 - 11x + 10 &= 0 \\ (x - 10)(x - 1) &= 0 \\ x &= 1, 10 \end{aligned}$$

ii. (2 marks)

- ✓ [1] for each correct solution, i.e. $x = 3 \pm 4$.
- ✓ [1] for correct simplification of the solutions.

$$\begin{aligned} (x - 3)^2 &= 16 \\ x - 3 &= \pm 4 \\ x = 3 \pm 4 &= -1 \text{ or } 7 \end{aligned}$$

iii. (3 marks)

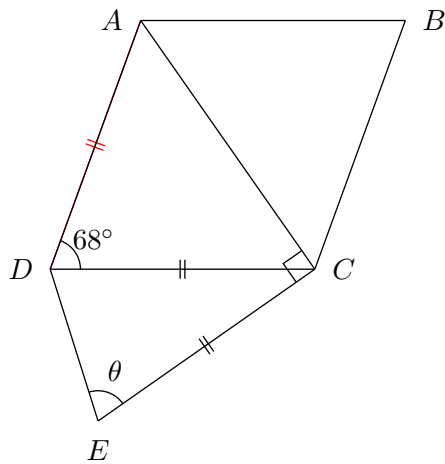
- ✓ [1] for completing the square.
- ✓ [1] for each correct solution.

$$\begin{aligned}
 x^2 + 6x + 9 - 9 - 14 &= 0 \\
 (x + 3)^2 - 23 &= 0 \\
 (x + 3)^2 &= 23 \\
 x + 3 &= \pm\sqrt{23} \\
 x &= -3 \pm \sqrt{23}
 \end{aligned}$$

Question 4

(a) (3 marks)

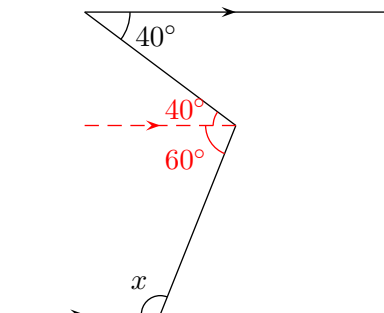
- ✓ [1] for correct calculations
- ✓ [1] for proper reasoning
- ✓ [1] for final answer



- $\angle DAC = \angle DCA = \frac{180^\circ - 68^\circ}{2} = 56^\circ$
(Base $\angle$ of isosceles $\triangle DAC$)
- $\therefore \angle DCE = 90^\circ - 56^\circ = 34^\circ$
(complementary)
- $\theta = \frac{180^\circ - \angle DCE}{2} = 73^\circ$
(Base $\angle$ of isosceles $\triangle DEC$)

(b) (3 marks)

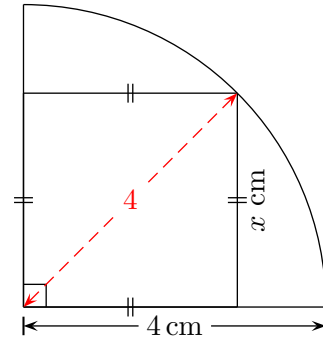
- ✓ [1] for construction of additional interval
- ✓ [1] for proper reasoning
- ✓ [1] for final answer



- Construct new line parallel to the other two horizontal lines.
- $x = 180^\circ - 60^\circ = 120^\circ$ (alternative $\angle$ on parallel lines)

(c) (3 marks)

- ✓ [1] for identifying radius as the hypotenuse.
- ✓ [1] for Pythagoras' Theorem.
- ✓ [1] for final answer.



Using Pythagoras' Theorem on the right triangle,

$$\begin{aligned}
 x^2 + x^2 &= 4^2 \\
 2x^2 &= 16 \\
 x^2 &= 8 \\
 x &= 2\sqrt{2}
 \end{aligned}$$

Question 5

(a) (2 marks)

✓ [1] for each correct value of m and b .

$$2x - 3y + 12 = 0$$

$$3y = 2x + 12$$

$$y = \frac{2}{3}x + 4$$

$$m = \frac{2}{3} \quad b = 4$$

(b) i. (3 marks)

✓ [1] for finding the gradient of AB .✓ [1] for correct y intercept.

✓ [1] for general form.

$$\frac{y+3}{x+1} = \frac{2+3}{3+1} = \frac{5}{4}$$

$$4y + 12 = 5x + 5$$

$$5x - 4y - 7 = 0$$

$$y = \frac{5}{4}x - \frac{7}{4}$$

$$5x - 4y - 7 = 0$$

ii. (1 mark)

$$m_{\perp} = -\frac{4}{5}$$

iii. (2 marks)

✓ [1] for substantial working to final answer.

✓ [1] for final answer.

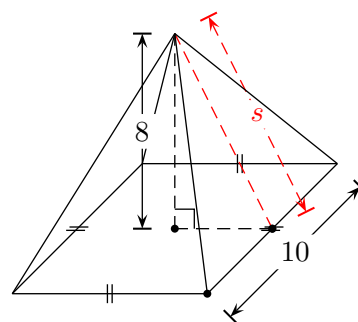
$$\frac{y-2}{x-3} = -\frac{4}{5}$$

$$y - 2 = -\frac{4}{5}x + \frac{12}{5}$$

$$y = -\frac{4}{5}x + \frac{22}{5}$$

Question 6

(a) i. (1 mark)



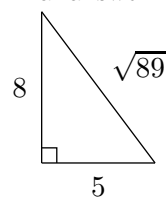
$$V = \frac{1}{3}Ah = \frac{1}{3} \times 10^2 \times 8 = \frac{800}{3}$$

ii. (3 marks)

✓ [1] for area of one of the side triangles.

✓ [1] for area of the base of the pyramid.

✓ [1] for final answer.



$$s = \sqrt{8^2 + 5^2}$$

$$= \sqrt{89}$$

$$A_{\text{side triangle}} = \frac{1}{2}bh = \frac{1}{2}\sqrt{89} \times 10$$

$$= 5\sqrt{89}$$

$$SA = 4A_{\text{side triangle}} + A_{\text{base}}$$

$$= 20\sqrt{89} + 100$$

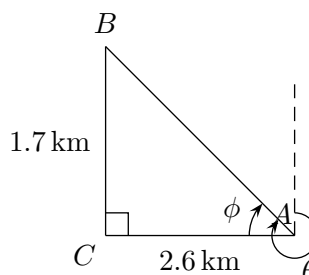
(b) (1 mark)

$$\sin \theta = \frac{8}{17}$$

(c) (2 marks)

✓ [1] for correct diagram.

✓ [1] for correct calculations, based on the diagram.



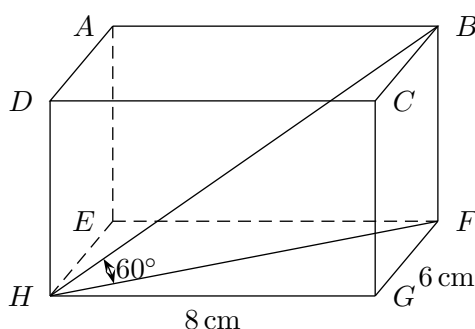
$$\tan \phi = \frac{1.7}{2.6}$$

$$\phi = 33.1785 \dots$$

$$\therefore \theta = 270^\circ + 33.1785 \dots \approx 303^\circ 11'$$

(d) i. (2 marks)

- ✓ [1] for applying the tan ratio.
- ✓ [1] for final answer.



$$\tan \angle FHG = \frac{6}{8}$$

$$\angle FHG = 37^\circ$$

(nearest degree)

ii. (1 mark)

$$FH = \sqrt{6^2 + 8^2} = 10 \text{ cm}$$

iii. (2 marks)

- ✓ [1] for applying the cosine ratio.
- ✓ [1] for final answer.

$$\cos 60^\circ = \frac{10}{BH}$$

$$BH = \frac{10}{\cos 60^\circ} = 20 \text{ cm}$$

iv. (2 marks)

- ✓ [1] for applying Pythagoras' Theorem.
- ✓ [1] for final answer.

$$BF^2 = 20^2 - 10^2$$

$$\therefore BF = \sqrt{300} = 10\sqrt{3} \text{ cm}$$

Question 7

(a) (3 marks)

- ✓ [1] for correctly multiplying up by 5 and 2 for equations (1) and (2) respectively.
- ✓ [1] for each correct value of x and y .

$$\begin{cases} 2x - 3y = -8 & (1) \\ 5x + 4y = 3 & (2) \end{cases}$$

$$(1) \times 5 \quad (2) \times 2$$

$$\begin{cases} 10x - 15y = -40 & (3) \\ 10x + 8y = 6 & (4) \end{cases}$$

$$(3) - (4) \Rightarrow -23y = -46$$

$$y = 2 \rightarrow (1)$$

$$2x - 6 = 8$$

$$2x = -2 \Rightarrow x = -1$$

$$\therefore x = -1 \quad y = 2$$

(b) (4 marks)

- ✓ [1] setting up pair of simultaneous equations.
- ✓ [1] for attempts to eliminate or substitute.
- ✓ [1] each for correct x and y .

Let x be the number of 2 mark questions,
 y be the number of 3 mark questions.

$$\begin{cases} x + y = 20 & (1) \\ 2x + 3y = 52 & (2) \end{cases}$$

$$(1) \times 2,$$

$$2x + 2y = 40 \quad (3)$$

$$(2) - (3):$$

$$y = 12$$

Substitute to (1):

$$\therefore x + 12 = 20$$

$$x = 8$$

Question 8

- (a) i. (1 mark) - Geography
 ii. (1 mark) - Geography
 iii. (1 mark) - Maths
 iv. (1 mark) - Music

- (b) (2 marks)

- ✓ [1] for correct future value (amount).
 ✓ [1] for finding the interest earned.

$$A = 1\,500 \left(1 + \frac{0.015}{365} \right)^{3 \times 365}$$

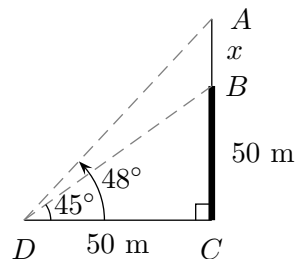
$$= \$1\,569.04$$

Interest earned: \$69.04

Question 9

- (a) (3 marks)

- ✓ [2] for obtaining $x = 50(\tan 48^\circ - 1) = 5.53$.
 ✓ [1] for rounding.



$$BC = 50 \text{ m}$$

$$\frac{AC}{50} = \frac{50 + x}{50} = \tan 48^\circ$$

$$50 + x = 50 \tan 48^\circ$$

$$\therefore x = 50(\tan 48^\circ - 1)$$

$$= 6 \text{ m (nearest m)}$$

- (b) (4 marks)

- ✓ [1] for Pythagoras' Theorem
 ✓ [1] for correct expansion
 ✓ [1] for $x^2 = 25$
 ✓ [1] for $x = \pm 5$

$$(1 + x^2)^2 = 10^2 + (x^2 - 1)^2$$

$$1 + 2x^2 + x^4 = 100 + x^4 - 2x^2 + 1$$

$$4x^2 = 100$$

$$x^2 = 25$$

$$x = \pm 5$$