

# MATHEMATICS

## Stage 5

### Statistics and Probability

### Probability

### Relative Frequency and Venn Diagrams

### Multi-Stage Events

Teacher: Haeun Jahg

Exam Equivalent Time: 117 minutes (based on allocation ~1.25 min/mark)

Questions

1. Relative Frequency and Venn Diagrams (010126)

A group of 150 people was surveyed and the results recorded.

| Survey results |                       |                              |              |
|----------------|-----------------------|------------------------------|--------------|
|                | <i>Owens a mobile</i> | <i>Does not own a mobile</i> | <i>Total</i> |
| <i>Male</i>    | 42                    | 28                           | 70           |
| <i>Female</i>  | 63                    | 17                           | 80           |
|                | 105                   | 45                           | 150          |

A person is selected at random from the surveyed group.

What is the probability that the person selected is a male who does not own a mobile?

- A.

$\frac{28}{150}$
- B.

$\frac{45}{150}$
- C.

$\frac{28}{70}$
- D.

$\frac{45}{70}$

2. Relative Frequency and Venn Diagrams (015128)

Leanne copied a two-way table into her book.

|                       | <i>Male</i> | <i>Female</i> | <i>Totals</i> |
|-----------------------|-------------|---------------|---------------|
| <i>Full-time work</i> | 279         | 356           | 635           |
| <i>Part-time work</i> | 187         | 439           | 716           |
| <i>Totals</i>         | 466         | 885           | 1351          |

Leanne made an error in copying one of the values in the shaded section of the table.

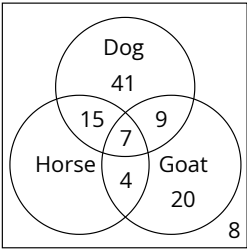
Which value has been incorrectly copied?

- A. The number of males in full-time work
- B. The number of males in part-time work
- C. The number of females in full-time work
- D. The number of females in part-time work

3. Relative Frequency and Venn Diagrams (251009)

A country school surveyed 120 of its students about the type of animals they have at home.

The results are recorded in the Venn diagram below, although the number of students who only own horses is missing.



If one of the students is selected at random, what is the probability that the student does not own a goat?

- A.  $\frac{40}{120}$
- B.  $\frac{60}{120}$
- C.  $\frac{80}{120}$
- D.  $\frac{100}{120}$

4. Relative Frequency and Venn Diagrams (251003)

The table below shows all the people at Angus' birthday party.

|       | Male | Female |
|-------|------|--------|
| Adult | 15   | 25     |
| Child | 40   | 20     |

What fraction of the children at the party are female?

- A.  $\frac{20}{60}$
- B.  $\frac{45}{60}$
- C.  $\frac{20}{100}$
- D.  $\frac{45}{100}$

5. Relative Frequency and Venn Diagrams (025668)

On a television game show, viewers voted for their favourite contestant. The results were recorded in the two-way table.

|              | Male viewers | Female viewers |
|--------------|--------------|----------------|
| Contestant 1 | 1372         | 3915           |
| Contestant 2 | 2054         | 3269           |

One male viewer was selected at random from all of the male viewers.

What is the probability that he voted for Contestant 1?

- A.  $\frac{1372}{10\,610}$
- B.  $\frac{1372}{5287}$
- C.  $\frac{1372}{3426}$
- D.  $\frac{1372}{2054}$

6. Relative Frequency and Venn Diagrams (060354)

A group of 485 people was surveyed. The people were asked whether or not they smoke. The results are recorded in the table.

|        | Smokers | Non-smokers | Total |
|--------|---------|-------------|-------|
| Male   | 88      | 176         | 264   |
| Female | 68      | 153         | 221   |
|        | 156     | 329         | 485   |

A person is selected at random from the group.

What is the approximate probability that the person selected is a smoker OR is male?

- A. 33%
- B. 18%
- C. 68%
- D. 87%

7. Multi-Stage Events (250871)

Bromley rolls two standard dice at the same time and adds up the total.

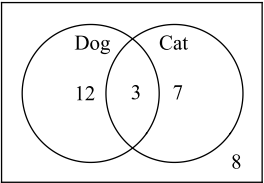
An incomplete table of the possible outcomes is below.

| Face | 1 | 2 | 3 | 4  | 5  | 6  |
|------|---|---|---|----|----|----|
| 1    | 2 | 3 | 4 | 5  | 6  | 7  |
| 2    | 3 | 4 | 5 |    |    | 8  |
| 3    | 4 | 5 | 6 |    |    | 9  |
| 4    | 5 |   | 7 | 8  | 9  | 10 |
| 5    | 6 |   | 8 |    | 10 | 11 |
| 6    | 7 | 8 | 9 | 10 | 11 | 12 |

- i. Complete the table. (2 marks)
- ii. What is the most likely total that Bromley will roll? (1 mark)

8. Relative Frequency and Venn Diagrams (251070)

A class of 30 students were surveyed about their pets. They were asked whether they owned a dog, cat, both or neither and the results were recorded in the Venn diagram below.

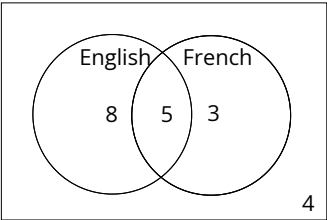


Record this information in the partially completed 2-way table below. (2 marks)

|        | Dog | No Dog |
|--------|-----|--------|
| Cat    |     |        |
| No Cat |     | 8      |

9. Relative Frequency and Venn Diagrams (251080)

A group of 20 museum visitors were surveyed about what languages they could speak fluently. They were surveyed on whether they could speak English or French and the results were recorded in the Venn diagram below.



Record this information in the partially completed 2-way table below. (2 marks)

|           | English | No English |
|-----------|---------|------------|
| French    |         |            |
| No French | 8       |            |

10. Multi-Stage Events (025595)

There are 100 tickets sold in a raffle. Justine sold all 100 tickets to five of her friends. The number of tickets she sold to each friend is shown in the table.

| Friend   | Number of tickets |
|----------|-------------------|
| Danielle | 45                |
| Khalid   | 5                 |
| Nancy    | 10                |
| Shani    | 14                |
| Herman   | 26                |
| Total    | 100               |

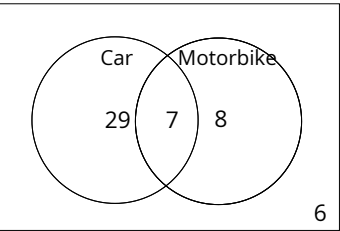
- a. Justine claims that each of her friends is equally likely to win first prize.  
Give a reason why Justine's statement is NOT correct. (1 mark)
- b. What is the probability that first prize is NOT won by Khalid or Herman? (2 marks)

11. Multi-Stage Events (250895)

Two fair 20 cent coins are tossed at the same time.  
What is the probability that both coins will show heads? (2 marks)

12. Relative Frequency and Venn Diagrams (251084)

A group of coalminers were surveyed about what registered vehicles they own. They were surveyed on whether they own a car, a motorbike, both or neither and the results were recorded in the Venn diagram below.

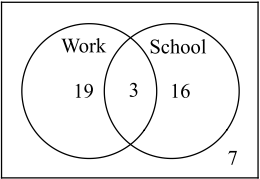


Record this information in the partially completed 2-way table below. (2 marks)

|              | Car | No Car |
|--------------|-----|--------|
| Motorbike    |     | 8      |
| No Motorbike |     |        |

13. Relative Frequency and Venn Diagrams (251016)

Zilda took a survey of eighteen year olds, asking if they work, go to school, do both or do neither.  
The Venn diagram shows the results.



What is the probability that a person randomly selected from the group goes to school and works, rounded to three decimal places? (2 marks)

14. Relative Frequency and Venn Diagrams (250997)

A group of 125 people were asked if they wear a watch or not.  
This table shows the results.

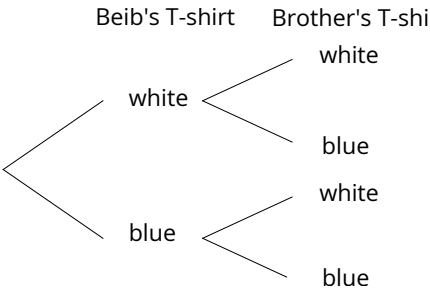
|       | Wears Watch | No Watch | Total |
|-------|-------------|----------|-------|
| Men   | 25          | 35       | 60    |
| Women | 15          | 50       | 65    |
| Total | 40          | 85       | 125   |

A man was selected at random.  
What is the exact probability that he wears a watch? (2 marks)

15. Multi-Stage Events (250950)

Beib owns three white and five blue T-shirts. He chooses a T-shirt at random for himself and puts it on. He then chooses another T-shirt at random, from the remaining T-shirts, and gives it to his brother.

- i. What is the probability that Beib chooses a blue T-shirt for himself? (1 mark)
- ii. Complete the tree diagram by writing the correct probability on each branch. (2 marks)



- iii. Calculate the probability that both of the T-shirts are the same colour. (2 marks)

16. Relative Frequency and Venn Diagrams (026939)

A new test has been developed for determining whether or not people are carriers of the Gaussian virus.  
Two hundred people are tested. A two-way table is being used to record the results.

|               | Positive | Negative |
|---------------|----------|----------|
| Carrier       | 74       | 12       |
| Not a carrier | 16       | A        |

- a. What is the value of A? (1 mark)
- b. A person selected from the tested group is a carrier of the virus.  
What is the probability that the test results would show this? (2 marks)
- c. For how many of the people tested were their test results inaccurate? (1 mark)

17. Relative Frequency and Venn Diagrams (014420)

Cecil invited 175 movie critics to preview his new movie. After seeing the movie, he conducted a survey. Cecil has almost completed the two-way table.

|                                          | <i>Aged &lt;40</i> | <i>Aged ≥40</i> | <i>Totals</i> |
|------------------------------------------|--------------------|-----------------|---------------|
| Movie critics who liked the movie        | 65                 |                 | 102           |
| Movie critics who did not like the movie |                    | 31              |               |
| <i>Totals</i>                            |                    | <b>A</b>        | 175           |

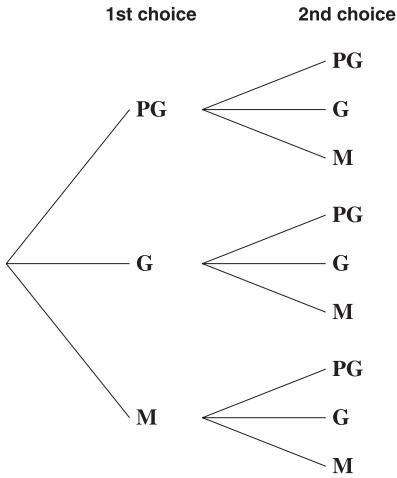
- a. Determine the value of *A*. (1 mark)
- b. A movie critic is selected at random.  
What is the probability that the critic was less than 40 years old and did not like the movie? (2 marks)
- c. Cecil believes that his movie will be a box office success if 65% of the critics who were surveyed liked the movie.  
Will this movie be considered a box office success? Justify your answer. (1 mark)

18. Multi-Stage Events (020615)

In a stack of 10 DVDs, there are 5 rated PG, 3 rated G and 2 rated M.

- a. A DVD is selected at random. What is the probability that it is rated M? (1 mark)

Grant chooses two DVDs at random from the stack. Copy or trace the tree diagram into your writing booklet.



- b. Complete the tree diagram by writing the correct probability on each branch. (2 marks)
- c. Calculate the probability that Grant chooses two DVDs with the same rating. (2 marks)

19. Multi-Stage Events (250917)

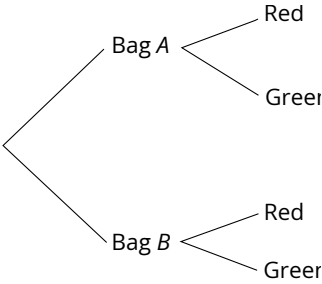
Tay-Tay has 2 bags of apples.

Bag A contains 4 red apples and 3 green apples.

Bag B contains 3 red apples and 1 green apple.

Tay-Tay chooses one of the bags randomly and with her eyes closed, takes one of the apples.

- i. Complete the tree diagram below. (2 marks)



- ii. Determine the probability that Tay-Tay chooses a red apple? (2 marks)

20. Multi-Stage Events (250802)

A coin is tossed 3 times. There are 8 possible outcomes.

What is the probability of getting 2 heads and 1 tail in any order? (2 marks)

21. Multi-Stage Events (250926)

Two dice are rolled. What is the probability that only one of the dice shows a three? (2 marks)

22. Relative Frequency and Venn Diagrams (251041)

Some men and women were surveyed at a football game. They were asked which team they supported. The results are shown in the two-way table.

|        | <i>Team A</i> | <i>Team B</i> | <i>Totals</i> |
|--------|---------------|---------------|---------------|
| Men    | 125           | 100           | 225           |
| Women  | 75            | 90            | 165           |
| Totals | 200           | 190           | 390           |

A man was chosen at random. What is the probability that he supports *Team B*, correct to the nearest percent? (2 marks)

23. Relative Frequency and Venn Diagrams (026272)

Lie detector tests are not always accurate. A lie detector test was administered to 200 people.

The results were:

- 50 people lied. Of these, the test indicated that 40 had lied;
- 150 people did NOT lie. Of these, the test indicated that 20 had lied.

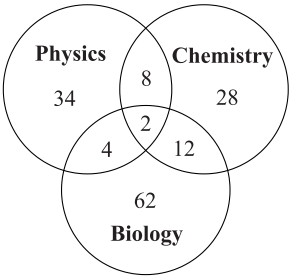
a. Complete the table using the information above. (2 marks)

|                           | Test indicated<br>a lie | Test did not<br>indicate a lie | Total |
|---------------------------|-------------------------|--------------------------------|-------|
| People<br>who lied        |                         |                                | 50    |
| People who<br>did NOT lie |                         |                                | 150   |

- b. For how many of the people tested was the lie detector test accurate? (1 mark)
- c. For what percentage of the people tested was the test accurate? (1 mark)
- d. What is the probability that the test indicated a lie for a person who did NOT lie? (1 mark)

24. Relative Frequency and Venn Diagrams (251028)

The subject choices in science at a high school are physics, chemistry and biology.  
This Venn diagram shows the number of students who are studying each of the subjects.

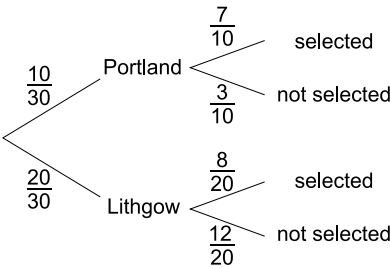


A student studying Biology is chosen a random.  
What is the probability that the student also studies Chemistry? (2 marks)

25. Multi-Stage Events (250810)

A representative soccer team is chosen from 30 players who play for two clubs, Portland and Lithgow.

10 players from Portland and 20 players from Lithgow are playing in the trials, and 7 players from Portland and 8 from Lithgow are selected in the representative team.



One player at the trial is randomly selected.  
What is the probability that the player is from Lithgow and is selected in the representative team?  
Give your answer to two decimal places. (2 marks)

26. Relative Frequency and Venn Diagrams (251096)

Students studying vocational education courses were surveyed about their living arrangements.

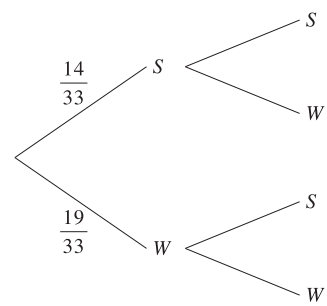
|                           | Females | Males | Totals |
|---------------------------|---------|-------|--------|
| Living with parent(s)     | 46      | 155   | 201    |
| Not living with parent(s) | 182     | 122   | 304    |
| Totals                    | 228     | 277   | 505    |

- i. One of these students is selected at random. What is the probability, correct to the nearest percentage, that this student is male and living with his parent(s)? (2 marks)
- ii. What is the probability that 'not living with her parent(s)' among selected females? (2 marks)

27. Multi-Stage Events (003169)

A box contains 33 scarves made from two different fabrics. There are 14 scarves made from silk (*S*) and 19 made from wool (*W*).  
Two girls each select, at random, a scarf to wear from the box.

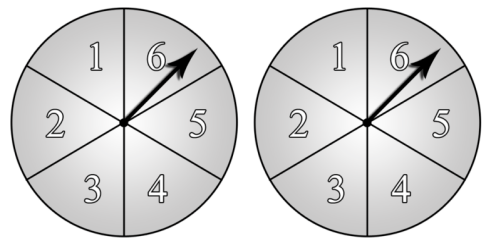
a. Complete the probability tree diagram below. (2 marks)



- b. Calculate the probability that the two scarves selected are made from silk. (1 mark)
- c. Calculate the probability that the two scarves selected are made from different fabrics. (2 marks)

28. Multi-Stage Events (250900)

Jon spins each pointer 50 times.



Each time he added the numbers that the pointers landed on.  
His results are shown below.

| Sum of numbers  | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | Total |
|-----------------|---|---|---|---|---|---|---|---|----|----|----|-------|
| Number of spins | 1 | 2 | 3 | 6 | 8 | 9 | 7 | 5 | 4  | 4  | 1  | 50    |

What percentage of the spins resulted in a sum of 9? (2 marks)

29. Multi-Stage Events (250913)

Each time she throws a dart, the probability that Gaga hits the dartboard is  $\frac{4}{7}$ .  
She throws two darts, one after the other.  
What is the probability that she misses the dartboard with both darts? (2 marks)

30. Multi-Stage Events (137090)

A bowl of fruit contains 17 apples of which 9 are red and 8 are green.  
Dennis takes one apple at random and eats it. Margaret also takes an apple at random and eats it.  
By drawing a probability tree diagram, or otherwise, find the probability that Dennis and Margaret eat apples of the same colour. (3 marks)

31. Multi-Stage Events (004644)

- The probability that Michael will score more than 100 points in a game of bowling is  $\frac{31}{40}$ .
- a. A commentator states that the probability that Michael will score less than 100 points in a game of bowling is  $\frac{9}{40}$ .  
Is the commentator correct? Give a reason for your answer. (1 mark)
- b. Michael plays two games of bowling. What is the probability that he scores more than 100 points in the first game and then again in the second game? (1 mark)

# Probability QB Answers

Q1.  $P = \frac{28}{150}$

Q2. The number of females in part-time work.

Q3.  $P = \frac{80}{120}$

Q4.  $P = \frac{20}{60}$

Q5.  $P = \frac{1372}{3426}$

Q6.  $P = \frac{264 + 68}{485} \approx 68\%$

Q7.

| Face | 1 | 2 | 3 | 4  | 5  | 6  |
|------|---|---|---|----|----|----|
| 1    | 2 | 3 | 4 | 5  | 6  | 7  |
| 2    | 3 | 4 | 5 | 6  | 7  | 8  |
| 3    | 4 | 5 | 6 | 7  | 8  | 9  |
| 4    | 5 | 6 | 7 | 8  | 9  | 10 |
| 5    | 6 | 7 | 8 | 9  | 10 | 11 |
| 6    | 7 | 8 | 9 | 10 | 11 | 12 |

ii. 7 is the most likely to be rolled.

Q8.

|        | Dog | No Dog |
|--------|-----|--------|
| Cat    | 3   | 7      |
| No Cat | 12  | 8      |

Q9.

|           | English | No English |
|-----------|---------|------------|
| French    | 5       | 3          |
| No French | 8       | 4          |

Q10. a) The claim is incorrect because each of her friends bought a different number of tickets and therefore their chances of winning are different.

b)  $P(\text{first prize is NOT won by K or H}) = \frac{69}{100}$

Q11.  $P(HH) = \frac{1}{4}$

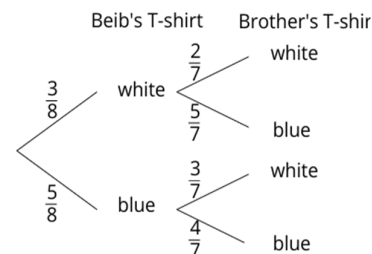
Q12.

|              | Car | No Car |
|--------------|-----|--------|
| Motorbike    | 7   | 8      |
| No Motorbike | 29  | 6      |

Q13.  $P(\text{go to school and works}) = \frac{3}{45} \approx 0.067$

Q14.  $P(\text{man chosen wears watch}) = \frac{5}{12}$

Q15. i.  $P(B) = \frac{5}{8}$



iii.  $P(\text{same colour}) = \frac{13}{28}$

# Probability QB Answers

Q16. a)  $A = 98$

b)  $P = \frac{37}{43}$

c) Number with inaccurate results = 28

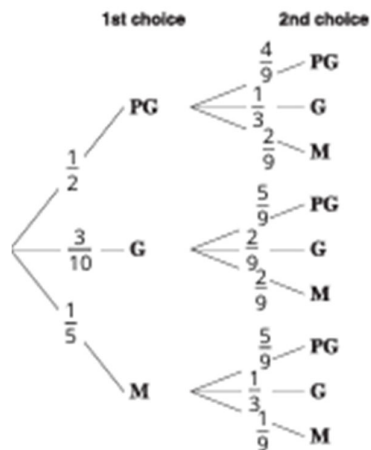
Q17. a)  $A = 68$

b)  $P(\text{not like and } < 40) = \frac{6}{25}$

c)  $P(\text{critics liked}) = \frac{102}{175} = 58.28 \dots \% \rightarrow$   
*not a box office success.*

Q18. a)  $P(M) = \frac{1}{5}$

b)



c)  $P(\text{same rating}) = \frac{14}{45}$

Q19. i.



ii.  $P(\text{red}) = \frac{37}{56}$

## Probability QB Answers

Q20. Strategy 1

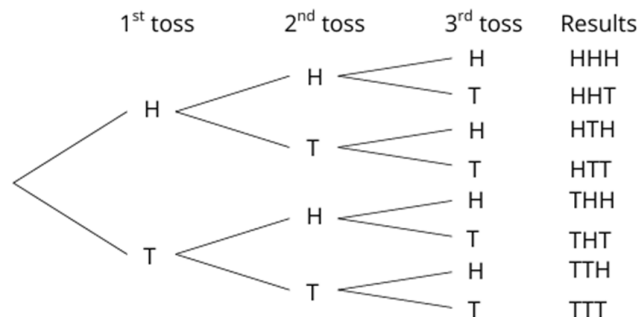
The table (array) below lists the possible outcomes:

|                 |   |   |   |   |   |   |   |   |
|-----------------|---|---|---|---|---|---|---|---|
| <b>1st toss</b> | H | H | H | H | T | T | T | T |
| <b>2nd toss</b> | H | H | T | T | H | H | T | T |
| <b>3rd toss</b> | H | T | H | T | H | T | H | T |
|                 |   |   |   |   |   |   |   |   |

From table,  $P(2H, 1T) = \frac{3}{8}$

Strategy 2

Using a probability tree:



Q21.  $P(\text{only one 3}) = \frac{5}{18}$

Q22.  $P(\text{chosen man supports Team B}) = \frac{4}{9} \approx 44\%$

Q23. a)

|                        | <i>Test indicated a lie</i> | <i>Test did not indicate a lie</i> | <i>Total</i> |
|------------------------|-----------------------------|------------------------------------|--------------|
| People who lied        | 40                          | 10                                 | 50           |
| People who did NOT lie | 20                          | 130                                | 150          |

b) 170

c) 85%

d)  $P(\text{lie detected when NOT a lie}) = \frac{2}{15}$

Q24.  $P(\text{chosen student studies Chemistry}) = \frac{7}{40}$

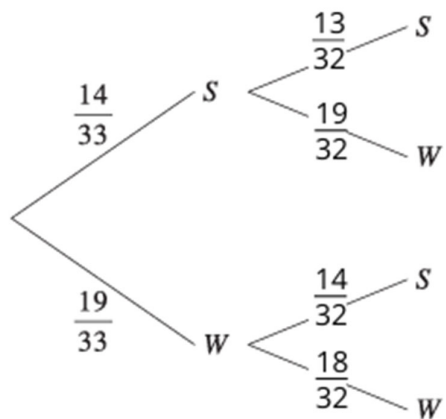
Q25.  $P(\text{player is from Lithgow and selected}) = \frac{8}{30} \approx 0.27$

Q26. i.  $P(\text{male living with parents}) = \frac{155}{505} \approx 31\%$

ii.  $P(\text{selected female not living with parents}) = \frac{91}{114}$

Probability QB Answers

Q27. a)



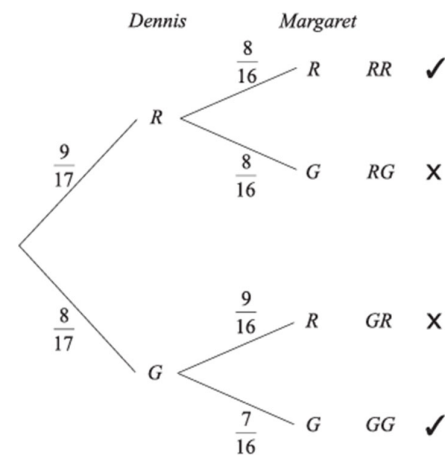
b)  $P(2 \text{ silk}) = \frac{91}{528}$

c)  $P(\text{different}) = \frac{133}{264}$

Q28. 10%

Q29.  $P(\text{misses twice}) = \frac{9}{49}$

Q30.  $P(\text{same colour}) = \frac{8}{17}$



Q31. a) The commentator is incorrect. The correct statement is:

$$P(\text{score} \leq 100) = \frac{9}{40}$$

b)  $P(\text{score} > 100 \text{ in both}) = \frac{961}{1600}$