NWCC

Algebraic Fractions Practice Problem Set

Building up to Year 10 Advanced Level

LEVEL 1: SIMPLIFYING BY CANCELLING MONOMIALS [1PT]

Focus: Applying index laws and simplifying single-term fractions.

Question 1 **(1 mark)**

Simplify: $\frac{6x^2}{3x}$

Question 2 **(1 mark)**

Simplify: $\frac{12a^3b}{4ab^2}$

Question 3 **(1 mark)**

Simplify: $\frac{-20m^5n^2}{5m^2n^4}$

Question 4 **(1 mark)**

Simplify: $\frac{2x^2y \times 6xy^3}{4x^2y^2}$

Question 5 **(1 mark)**

Simplify: $\frac{15p^4q}{(3pq)^2}$

LEVEL 2: SIMPLIFYING BY BINOMIAL FACTORING [1PT]

Focus: Factoring out a Highest Common Factor (HCF) or using the difference of two squares before simplifying.

Question 6 **(2 marks)**

Simplify: $\dfrac{4x+8}{4}$

Question 7 **(2 marks)**

Simplify: $\dfrac{3a-9}{a^2-3a}$

Question 8 **(2 marks)**

Simplify: $\dfrac{x^2-25}{x-5}$

Question 9 **(2 marks)**

Simplify: $\dfrac{4m^2-9n^2}{2m+3n}$

Question 10 **(2 marks)**

Simplify: $\dfrac{2k-10}{25-k^2}$

LEVEL 3: MULTIPLICATION AND DIVISION [1PT]

Focus: Factoring all numerators and denominators completely, then cross-cancelling common factors.

Question 11 **(2 marks)**

Simplify: $\dfrac{2x}{5} \times \dfrac{15}{4x^3}$

Question 12 **(2 marks)**

Simplify: $\dfrac{y^2-4}{3} \div \dfrac{y+2}{9}$

Question 13 **(3 marks)**

Simplify: $\dfrac{x^2-x-6}{x^2-9} \times \dfrac{x+3}{x-3}$

Question 14 **(3 marks)**

Simplify: $\dfrac{2a^2+5a-3}{4a^2-1} \div \dfrac{a+3}{2a+1}$

Question 15 **(3 marks)**

Simplify: $\dfrac{3x-12}{x^2-16} \times \dfrac{x^2+8x+16}{6x+24}$

LEVEL 4: ADDITION AND SUBTRACTION [1PT]

Focus: Finding a lowest common denominator (LCD), expanding numerators, and collecting like terms.

Question 16 **(2 marks)**

Simplify: $\frac{x+1}{3} + \frac{x-2}{4}$

Question 17 **(2 marks)**

Simplify: $\frac{5}{2x} - \frac{3}{4x}$

Question 18 **(3 marks)**

Simplify: $\frac{3}{x-2} + \frac{4}{x+3}$

Question 19 **(3 marks)**

Simplify: $\frac{2}{x-1} - \frac{5}{1-x}$

Question 20 **(3 marks)**

Simplify: $\frac{x}{x+2} - \frac{2}{x-2}$

Question 21 **(3 marks)**

Simplify: $\dfrac{4}{x^2-9}+\dfrac{2}{x-3}$

Question 22 **(4 marks)**

Simplify: $\dfrac{3}{x^2+3x+2}-\dfrac{1}{x+1}$

LEVEL 5: ADVANCED & EXAM-STYLE CHALLENGES (YEAR 10 ADVANCED) [1PT]

Focus: Multi-step simplifying, simultaneous index configurations, nested grouping, and solving equations containing algebraic fractions.

Question 23 **(4 marks)**

Express as a single simplified fraction: $\dfrac{3x}{x-1} - \dfrac{2}{x} - 3$

Question 24 **(4 marks)**

Express as a single simplified fraction: $\dfrac{1}{x^2-x} - \dfrac{1}{x^2+x}$

Question 25 **(4 marks)**

Simplify completely: $\dfrac{2x^2-8}{x^2+2x-8} \div \dfrac{4x-8}{x^2-16}$

Question 26 **(4 marks)**

Simplify to a single fraction: $\dfrac{3}{x+2} - \dfrac{2x-1}{x^2-4} + \dfrac{1}{x-2}$

Question 27 **(3 marks)**

Solve for x: $\dfrac{4}{x-1} = \dfrac{3}{x+2}$

Question 28 **(4 marks)**

Solve for x: $\dfrac{x}{3} + \dfrac{2}{x} = \dfrac{7}{3}$

Question 29 **(4 marks)**

Solve for x: $\dfrac{2}{x-1} + \dfrac{3}{x+1} = 2$

Question 30 **(5 marks)**

Solve the following equation for exact values of x: $\dfrac{1}{x-2} - \dfrac{2}{x+3} = 1$

SOLUTIONS AND MARKING GUIDELINES [1PT]

Level 1 Solutions

Question 1: $\dfrac{6x^2}{3x} = 2x$

Question 2: $\dfrac{12a^3b}{4ab^2} = \dfrac{3a^2}{b}$

Question 3: $\dfrac{-20m^5n^2}{5m^2n^4} = -\dfrac{4m^3}{n^2}$

Question 4: $\dfrac{2x^2y \times 6xy^3}{4x^2y^2} = \dfrac{12x^3y^4}{4x^2y^2} = 3xy^2$

Question 5: $\dfrac{15p^4q}{(3pq)^2} = \dfrac{15p^4q}{9p^2q^2} = \dfrac{5p^2}{3q}$

Level 2 Solutions

Question 6: $\dfrac{4(x+2)}{4} = x+2$

Question 7: $\dfrac{3(a-3)}{a(a-3)} = \dfrac{3}{a}$

Question 8: $\dfrac{(x-5)(x+5)}{x-5} = x+5$

Question 9: $\dfrac{(2m-3n)(2m+3n)}{2m+3n} = 2m-3n$

Question 10: $\dfrac{2(k-5)}{(5-k)(5+k)} = \dfrac{-2(5-k)}{(5-k)(5+k)} = -\dfrac{2}{5+k}$

Level 3 Solutions

Question 11: $\dfrac{2x}{5} \times \dfrac{15}{4x^3} = \dfrac{30x}{20x^3} = \dfrac{3}{2x^2}$

Question 12: $\dfrac{(y-2)(y+2)}{3} \times \dfrac{9}{y+2} = 3(y-2) = 3y-6$

Question 13: $\dfrac{(x-3)(x+2)}{(x-3)(x+3)} \times \dfrac{x+3}{x-3} = \dfrac{x+2}{x-3}$

Question 14: $\dfrac{(2a-1)(a+3)}{(2a-1)(2a+1)} \times \dfrac{2a+1}{a+3} = 1$

Question 15: $\dfrac{3(x-4)}{(x-4)(x+4)} \times \dfrac{(x+4)^2}{6(x+4)} = \dfrac{3}{6} = \dfrac{1}{2}$

Level 4 Solutions

Question 16: $\dfrac{4(x+1)+3(x-2)}{12} = \dfrac{4x+4+3x-6}{12} = \dfrac{7x-2}{12}$

Question 17: $\dfrac{10}{4x} - \dfrac{3}{4x} = \dfrac{7}{4x}$

Question 18: $\dfrac{3(x+3)+4(x-2)}{(x-2)(x+3)} = \dfrac{3x+9+4x-8}{(x-2)(x+3)} = \dfrac{7x+1}{(x-2)(x+3)}$

Question 19: Note that $1-x = -(x-1)$, so $\dfrac{2}{x-1} - \dfrac{5}{-(x-1)} = \dfrac{2}{x-1} + \dfrac{5}{x-1} = \dfrac{7}{x-1}$

Question 20: $\dfrac{x(x-2)-2(x+2)}{(x+2)(x-2)} = \dfrac{x^2-2x-2x-4}{x^2-4} = \dfrac{x^2-4x-4}{x^2-4}$

Question 21: $\dfrac{4}{(x-3)(x+3)} + \dfrac{2(x+3)}{(x-3)(x+3)} = \dfrac{4+2x+6}{(x-3)(x+3)} = \dfrac{2x+10}{(x-3)(x+3)}$

Question 22: $\dfrac{3}{(x+1)(x+2)} - \dfrac{1(x+2)}{(x+1)(x+2)} = \dfrac{3-x-2}{(x+1)(x+2)} = \dfrac{1-x}{(x+1)(x+2)}$

Level 5 Solutions

Question 23: Convert to common denominator $x(x-1)$:
$$\frac{3x(x)-2(x-1)-3[x(x-1)]}{x(x-1)} = \frac{3x^2-2x+2-3[x^2-x]}{x(x-1)} = \frac{3x^2-2x+2-3x^2+3x}{x(x-1)} = \frac{x+2}{x(x-1)}$$

Question 24: $\dfrac{1}{x(x-1)} - \dfrac{1}{x(x+1)} = \dfrac{(x+1)-(x-1)}{x(x-1)(x+1)} = \dfrac{2}{x(x^2-1)}$

Question 25: $\dfrac{2(x-2)(x+2)}{(x+4)(x-2)} \times \dfrac{(x-4)(x+4)}{4(x-2)} = \dfrac{2(x+2)(x-4)}{4(x-2)} = \dfrac{(x+2)(x-4)}{2(x-2)}$

Question 26: Common denominator is $(x-2)(x+2) = x^2-4$:
$$\frac{3(x-2)-(2x-1)+1(x+2)}{x^2-4} = \frac{3x-6-2x+1+x+2}{x^2-4} = \frac{2x-3}{x^2-4}$$

Question 27: Cross-multiply: $4(x+2) = 3(x-1) \implies 4x+8 = 3x-3 \implies x = -11$

Question 28: Multiply by LCD $3x$: $x^2+6 = 7x \implies x^2-7x+6 = 0 \implies (x-1)(x-6) = 0$.
Solutions: $x = 1$ or $x = 6$

Question 29: Multiply by LCD $(x-1)(x+1)$: $2(x+1)+3(x-1) = 2(x-1)(x+1)$
$2x+2+3x-3 = 2(x^2-1) \implies 5x-1 = 2x^2-2 \implies 2x^2-5x-1 = 0$
Using the quadratic formula: $x = \dfrac{-(-5) \pm \sqrt{(-5)^2-4(2)(-1)}}{2(2)} = \dfrac{5 \pm \sqrt{33}}{4}$

Question 30: Multiply by LCD $(x-2)(x+3)$: $1(x+3)-2(x-2) = 1(x-2)(x+3)$
$x+3-2x+4 = x^2+x-6 \implies -x+7 = x^2+x-6 \implies x^2+2x-13 = 0$
Using the quadratic formula: $x = \dfrac{-2 \pm \sqrt{2^2-4(1)(-13)}}{2(1)} = \dfrac{-2 \pm \sqrt{56}}{2}$
Simplifying the radical: $\sqrt{56} = 2\sqrt{14}$, so $x = \dfrac{-2 \pm 2\sqrt{14}}{2} = -1 \pm \sqrt{14}$

— End of Revision Booklet —