ADVANCED (2027)

Stage 6

Functions (Adv-2027)
Working with Functions (Y11)
Quadratics and Cubic Functions

Teacher: Jana Thorrold
Exam Equivalent Time: 58.5 minutes *(based on allocation of 1.5 minutes per mark)*

Questions

1. Functions, 2ADV F1 2013 HSC 1 MC

RAP Data (vs state mean) - Bottom 21%: School result (50%) was 29% below state average (79%)

What are the solutions of $2x^2 - 5x - 1 = 0$?

A. $x = \dfrac{-5 \pm \sqrt{17}}{4}$

B. $x = \dfrac{5 \pm \sqrt{17}}{4}$

C. $x = \dfrac{-5 \pm \sqrt{33}}{4}$

D. $x = \dfrac{5 \pm \sqrt{33}}{4}$

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2. Functions, 2ADV F1 2014 HSC 6 MC

Which expression is a factorisation of $8x^3 + 27$?

A. $(2x - 3)\left(4x^2 + 12x - 9\right)$

B. $(2x + 3)\left(4x^2 - 12x + 9\right)$

C. $(2x - 3)\left(4x^2 + 6x - 9\right)$

D. $(2x + 3)\left(4x^2 - 6x + 9\right)$

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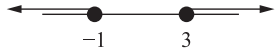
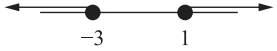
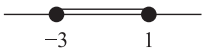
3. Functions, EXT1 F1 2020 HSC 1 MC

Which diagram best represents the solution set of $x^2 - 2x - 3 \geq 0$?

A.


B.


C.


D.


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4. Functions, 2ADV EQ-Bank 9 MC

The graphs of $y = mx + c$ and $y = ax^2$ will have no points of intersection for all values of m, c and a such that

A. $a > 0$ and $c > 0$

B. $m > 0$ and $c > 0$

C. $a > 0$ and $c > -\dfrac{m^2}{4a}$

D. $a < 0$ and $c > -\dfrac{m^2}{4a}$

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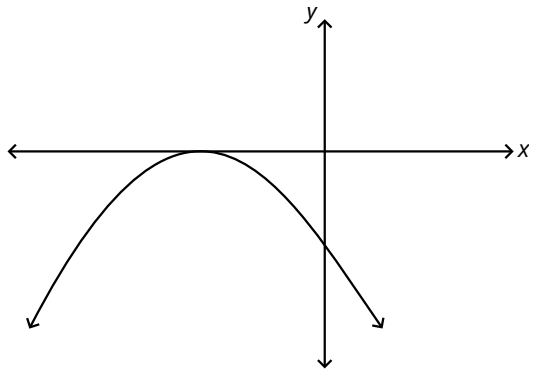
5. Functions, 2ADV F1 EQ-Bank 7 MC

The graph of a quadratic function $f(x) = ax^2 + bx + c$ is drawn below.



Which of the following are true?

A. $a < 0, c = 0$ and $b^2 - 4ac = 0$

B. $a > 0, c = 0$ and $b^2 - 4ac = 0$

C. $a > 0, c > 0$ and $b^2 - 4ac > 0$

D. $a < 0, c < 0$ and $b^2 - 4ac = 0$

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6. Functions, 2ADV EQ-Bank 8 MC

The equation $(p - 1)x^2 + 4x = 5 - p$ has no real roots when

A. $p^2 - 6p + 6 < 0$

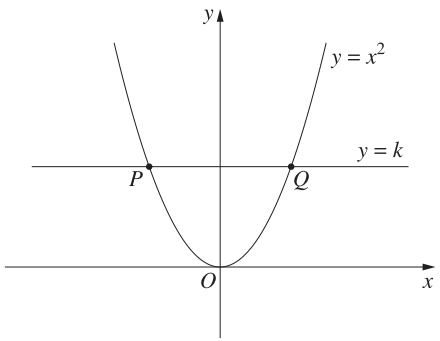
B. $p^2 - 6p + 1 > 0$

C. $p^2 - 6p - 6 < 0$

D. $p^2 - 6p + 1 < 0$

7. Functions, 2ADV F1 2023 HSC 10 MC

The graph $y = x^2$ meets the line $y = k$ (where $k > 0$) at points P and Q as shown in the diagram. The length of the interval PQ is L.



Let a be a positive number. The graph $y = \dfrac{x^2}{a^2}$ meets the line $y = k$ at points S and T.

What is the length of ST?

A. $\dfrac{L}{a}$

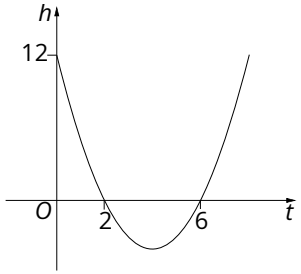
B. $\dfrac{L}{a^2}$

C. aL

D. a^2L

8. Functions, 2ADV F1 2025 HSC 11

The graph of a quadratic function represented by the equation $h = t^2 - 8t + 12$ is shown.



a. Find the values of t and h at the turning point of the graph. ***(2 marks)***

b. The graph shows $h = 12$ when $t = 0$.

What is the other value of t for which $h = 12$? ***(1 mark)***

9. Functions, 2ADV F1 2016 HSC 11e

Find the points of intersection of $y = -5 - 4x$ and $y = 3 - 2x - x^2$. *(3 marks)*

10. Functions, 2ADV F1 2010 HSC 1a

RAP Data (vs state mean) - Bottom 2%: School result (31%) was 50% below state average (81%)

Solve $x^2 = 4x$. *(2 marks)*

11. Functions, 2ADV F1 2015 HSC 11b

Factorise fully $3x^2 - 27$. *(2 marks)*

12. Functions, 2ADV EQ-Bank 11

Solve for x, giving your answers in the simplest form $a + b\sqrt{c}$ where a, b and c are real:

$5x^2 - 20x + 4 = 0$ *(2 marks)*

13. Functions, EXT1 F1 2024 HSC 11b

Solve $x^2 - 8x - 9 \leq 0$. ***(2 marks)***

14. Functions, EXT1 F1 EQ-Bank 12

Find the values of x that satisfy the equation

$x^2 + 8x + 3 \leq 0$. ***(3 marks)***

15. Functions, EXT1* F1 2010 HSC 2b

Solve the inequality $x^2 - x - 12 < 0$. ***(2 marks)***

16. Functions, 2ADV EQ-Bank 16

a. Express $y = x^2 - 4x + 6$ in the form $y = (x - a)^2 + c$. ***(1 mark)***

b. Graph the parabola, labelling its vertex and y-intercept. ***(2 marks)***

17. Functions, 2ADV 2006 HSC 7c

i. Write down the discriminant of $2x^2 + (k - 2)x + 8$ where k is a constant. ***(1 mark)***

ii. Hence, or otherwise, find the values of k for which the parabola $y = 2x^2 + kx + 9$ does not intersect the line $y = 2x + 1$. ***(2 marks)***

18. Functions, 2ADV EQ-Bank 17

Using the discriminant, or otherwise, justify why the graph of $f(x) = -x^2 + 2x - 2$ lies entirely below the x-axis. ***(2 marks)***

19. Functions, 2ADV F1 EQ-Bank 23

Find the values of k for which the expression $x^2 - 3x + (4 - 2k)$ is always positive. ***(3 marks)***

20. Functions, 2ADV F1 EQ-Bank 29

The tangent to the parabola $y = x^2 + 2x - 4$ is $y = px - 5$ where $p > 0$.

Find the value of p. ***(2 marks)***

Worked Solutions

1. Functions, 2ADV F1 2013 HSC 1 MC

$2x^2 - 5x - 1 = 0$

Using $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{5 \pm \sqrt{(-5)^2 - 4 \times 2 \times (-1)}}{2 \times 2}$$

$$= \frac{5 \pm \sqrt{25 + 8}}{4}$$

$$= \frac{5 \pm \sqrt{33}}{4}$$

$\Rightarrow D$

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2. Functions, 2ADV F1 2014 HSC 6 MC

$$8x^3 + 27$$
$$= (2x)^3 + 3^3$$
$$= (2x + 3)\left(4x^2 - 6x + 9\right)$$

$\Rightarrow D$

COMMENT: Factorising a cubic is only examinable with scaffolding, as provided here by expanding the answer options.

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3. Functions, EXT1 F1 2020 HSC 1 MC

$$x^2 - 2x - 3 \geq 0$$
$$(x - 3)(x + 1) \geq 0$$

$x \geq 3$, or $x \leq -1$

$\Rightarrow A$

Worked Solutions

4. Functions, 2ADV EQ-Bank 9 MC

Intersect when:

$$mx + c = ax^2$$
$$ax^2 - mx - c = 0$$

Since no points of intersection:

$$\Delta < 0$$
$$m^2 - 4a(-c) < 0$$
$$m^2 + 4ac < 0$$

Solve for c:

$\therefore\ c > \dfrac{-m^2}{4a},\ \ a < 0$

or

$c < \dfrac{-m^2}{4a},\ \ a > 0$

$\Rightarrow D$

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5. Functions, 2ADV F1 EQ-Bank 7 MC

Quadratic touches x-axis once only $\Rightarrow b^2 - 4ac = 0$ (eliminate C)

Quadratic is inverted $\Rightarrow a < 0$ (eliminate B)

If $c = 0$, $f(x) = ax^2 + bx + 0 = x(ax + b) \Rightarrow$ cuts twice (Eliminate A)

$\Rightarrow D$

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6. Functions, 2ADV EQ-Bank 8 MC

$$(p - 1)x^2 + 4x + (p - 5) = 0$$

No real solutions when $\Delta < 0$:

$$b^2 - 4ac < 0$$
$$4^2 - 4(p - 1)(p - 5) < 0$$
$$16 - 4\left(p^2 - 6p + 5\right) < 0$$
$$-4p^2 + 24p - 4 < 0$$
$$p^2 - 6p + 1 > 0$$

$\Rightarrow B$

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7. Functions, 2ADV F1 2023 HSC 10 MC

Mean mark 24%.

Intersection of $y = x^2$ and $y = k$:

$x^2 = k \Rightarrow x = \pm\sqrt{k}$

$\therefore L = 2\sqrt{k}$

Intersection of $y = \dfrac{x^2}{a^2}$ and $y = k$:

$\dfrac{x^2}{a^2} = k$

$x^2 = a^2k$

$x = \pm a\sqrt{k}$

$\therefore ST = a \times 2\sqrt{k} = aL$

$\Rightarrow C$

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8. Functions, 2ADV F1 2025 HSC 11

a. Strategy 1 (no calculus)

Axis of quadratic occurs when $t = \dfrac{2+6}{2} = 4$

At $t = 4$:

$h = 4^2 - 8 \times 4 + 12 = -4$

$\therefore$ Turning point at $(4, -4)$

Strategy 2 (using calculus)

$h = t^2 - 8t + 12$

$h' = 2t - 8$

Find t when $h' = 0$:

$2t - 8 = 0 \Rightarrow t = 4$

b. When $h = 12$:

$t^2 - 8t + 12 = 12$

$t(t - 8) = 0$

$\therefore$ Other value: $t = 8$

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9. Functions, 2ADV F1 2016 HSC 11e

$y = 3 - 2x - x^2$

Substitute $y = -5 - 4x$ into equation

$-5 - 4x = 3 - 2x - x^2$

$x^2 - 2x - 8 = 0$

$(x - 4)(x + 2) = 0$

$\therefore x = 4$ or -2

When $x = 4$, $y = -5 - 4(4) = -21$

When $x = -2$, $y = -5 - 4(-2) = 3$

$\therefore$ Intersection at $(4, -21)$ and $(-2, 3)$

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10. Functions, 2ADV F1 2010 HSC 1a

$x^2 = 4x$

$x^2 - 4x = 0$

$x(x - 4) = 0$

$\therefore x = 0$ or 4

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11. Functions, 2ADV F1 2015 HSC 11b

$3x^2 - 27 = 3\left(x^2 - 9\right)$

$= 3(x + 3)(x - 3)$

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12. Functions, 2ADV EQ-Bank 11

$5x^2 - 20x + 4 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{20 \pm \sqrt{20^2 - 4 \times 5 \times 4}}{2 \times 5}$

$= \dfrac{20 \pm \sqrt{320}}{10}$

$= 2 \pm \dfrac{8\sqrt{5}}{10}$

$= 2 \pm \dfrac{4}{5}\sqrt{5}$

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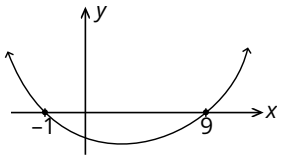
13. Functions, EXT1 F1 2024 HSC 11b

$$x^2 - 8x - 9 \leqslant 0$$
$$(x-9)(x+1) \leqslant 0$$



$$\therefore -1 \leqslant x \leqslant 9$$

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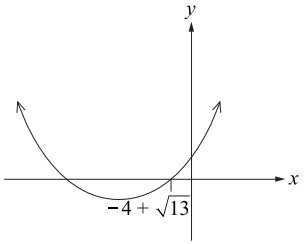
14. Functions, EXT1 F1 EQ-Bank 12

$$x = \frac{-8 \pm \sqrt{8^2 - 4 \cdot 1 \cdot 3}}{2}$$
$$= \frac{-8 \pm \sqrt{52}}{2}$$
$$= -4 \pm \sqrt{13}$$



$$\left\{x: \ -4 - \sqrt{13} \leq x \leq -4 + \sqrt{13}\right\}$$

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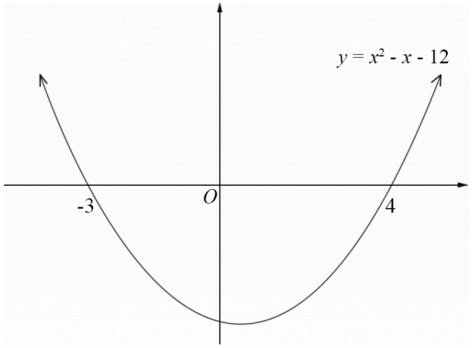
15. Functions, EXT1* F1 2010 HSC 2b

$$x^2 - x - 12 < 0$$
$$x^2 - x - 12 = 0$$
$$(x-4)(x+3) = 0$$
$$\Rightarrow x = 4 \text{ or } -3$$

MARKER'S COMMENT: Drawing the parabola proved the most efficient solution strategy.



$$x^2 - x - 12 < 0 \ \text{ when } \ -3 < x < 4$$

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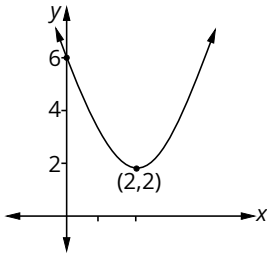
16. Functions, 2ADV EQ-Bank 16

a. $y = x^2 - 4x + 6$
$= x^2 - 4x + 4 + 2$
$= (x-2)^2 + 2$

b. Vertex $= (2, 2)$
y-intercept $= (0, 6)$



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17. Functions, 2ADV 2006 HSC 7c

i. $2x^2 + (k-2)x + 8$

$$\begin{aligned}\Delta &= b^2 - 4ac \\ &= (k-2)^2 - 4 \times 2 \times 8 \\ &= k^2 - 4k + 4 - 64 \\ &= k^2 - 4k - 60\end{aligned}$$

ii. $y = 2x^2 + kx + 9 \quad \ldots (1)$

$y = 2x + 1 \qquad \ldots (2)$

Substitute $y = 2x + 1$ into (1)

$2x + 1 = 2x^2 + kx + 9$

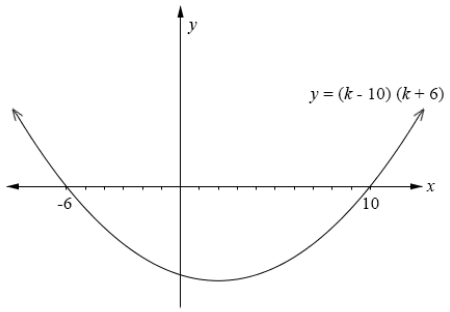
$2x^2 + kx - 2x + 8 = 0$

$2x^2 + (k-2)x + 8 = 0 \ldots (*)$

The graphs will not intercept if $(*)$ has no roots, i.e.)\ \ Delta <0

$$k^2 - 4k - 60 < 0$$

$$(k-10)(k+6) < 0$$



From the graph, no intersection when

$-6 < k < 10$

18. Functions, 2ADV EQ-Bank 17

$\Delta = b^2 - 4ac = 2^2 - 4(-1)(-2) = -4$

Since $\Delta < 0$, $y = -x^2 + 2x - 2$ does not intersect the x-axis.

Since $a = -1 < 0$, $f(x)$ is an upside down parabola.

$\Rightarrow f(x)$ must lie entirely below x-axis.

19. Functions, 2ADV F1 EQ-Bank 23

$x^2 - 3x + (4 - 2k) > 0$

$x^2 - 3x + (4 - 2k) = 0$ is a concave up parabola

$\Rightarrow$ Always positive (no roots) if $\Delta < 0$

$$\begin{aligned}b^2 - 4ac &< 0 \\ (-3)^2 - 4 \cdot 1 \cdot (4 - 2k) &< 0 \\ 9 - 16 + 8k &< 0 \\ 8k &< 7 \\ k &< \frac{7}{8}\end{aligned}$$

20. Functions, 2ADV F1 EQ-Bank 29

Intersection occurs when:

$$x^2 + 2x - 4 = px - 5$$

$$x^2 + (2-p)x + 1 = 0$$

COMMENT: Key is to recognise this is a discriminant question, not a calculus application.

Tangent touches once $\Rightarrow$ Discriminant $\Delta = 0$

$$\begin{aligned}(2-p)^2 - 4 \times 1 \times 1 &= 0 \\ 4 - 4p + p^2 - 4 &= 0 \\ p(p-4) &= 0 \\ p = 4 \quad (p > 0)\end{aligned}$$