NWCC Year 10 Mathematics Advanced
Half-Yearly Revision Booklet Solutions

Solutions Guide (2026)

Contents

1 SECTION A: INDICES AND SURDS

1.1 A1 - Index Laws

Question A1.1 Simplify each of the following, leaving answers with positive indices:

1. $m^7 \times m^3 \div m^4$

   Solution:
   Using the index laws ($m^a \times m^b = m^{a+b}$ and $m^a \div m^b = m^{a-b}$):
   $$m^{7+3-4} = m^6$$

2. $(3a^2b)^3$

   Solution:
   Distribute the power of 3 to every term inside the parentheses ($(xy)^a = x^a y^a$):
   $$3^3 \times (a^2)^3 \times b^3 = 27a^6b^3$$

3. $(4x^3y^{-2}) \div (2x^{-1}y^3)$

   Solution:
   Divide coefficients and subtract the indices of matching variables:
   $$\frac{4}{2} \cdot x^{3-(-1)} \cdot y^{-2-3} = 2x^4y^{-5}$$
   Expressing with a positive index:
   $$\frac{2x^4}{y^5}$$

Question A1.2 Simplify, expressing with positive indices:

$$(5m^3n)^2 \times (2mn^2)^3 \div (10m^4n^5)$$

Solution:

1. Expand the powers:
$$(25m^6n^2) \times (8m^3n^6) \div (10m^4n^5)$$
2. Multiply the first two terms:
$$(25 \times 8) \cdot m^{6+3} \cdot n^{2+6} = 200m^9n^8$$
3. Divide by the final term:
$$\frac{200m^9n^8}{10m^4n^5} = 20m^{9-4}n^{8-5} = 20m^5n^3$$

Question A1.3 Solve for x:

$$2^{x-1} = 2^{2x-6}$$

Solution:

Since the bases are identical (2), equate the powers:

$$x - 1 = 2x - 6$$

Subtract x from both sides:

$$-1 = x - 6$$

Add 6 to both sides:

$$x = 5$$

Question A1.4 Simplify, expressing with positive indices:

$$(x^{-1})(x^{-2}-1)^{-1}$$

Solution:

1. Rewrite with positive indices internally:

$$\left(\frac{1}{x}\right)\left(\frac{1}{x^2}-1\right)^{-1}$$

2. Create a common denominator inside the second set of parentheses:

$$\left(\frac{1}{x}\right)\left(\frac{1-x^2}{x^2}\right)^{-1}$$

3. Apply the negative reciprocal power ($\left(\frac{a}{b}\right)^{-1}=\frac{b}{a}$):

$$\left(\frac{1}{x}\right)\left(\frac{x^2}{1-x^2}\right)$$

4. Multiply and cancel a factor of x:

$$\frac{x^2}{x(1-x^2)}=\frac{x}{1-x^2}$$

1.2 A2 - Surds

Question A2.1 Simplify each of the following:

1. $\sqrt{200} - 3\sqrt{8} + \sqrt{50}$

   Solution:
   Simplify each radical by finding perfect squares:

   - $\sqrt{200} = \sqrt{100 \times 2} = 10\sqrt{2}$
   - $3\sqrt{8} = 3\sqrt{4 \times 2} = 3(2\sqrt{2}) = 6\sqrt{2}$
   - $\sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2}$

   Combine like terms:
   $$10\sqrt{2} - 6\sqrt{2} + 5\sqrt{2} = 9\sqrt{2}$$

2. $\sqrt{m^5} + 4m^2\sqrt{m}$

   Solution:
   Simplify the first radical term:
   $$\sqrt{m^5} = \sqrt{m^4 \cdot m} = m^2\sqrt{m}$$
   Combine like terms:
   $$m^2\sqrt{m} + 4m^2\sqrt{m} = 5m^2\sqrt{m}$$

3. $\sqrt{75} \times \sqrt{12}$

   Solution:
   Simplify each surd first:
   $$\sqrt{75} = \sqrt{25 \times 3} = 5\sqrt{3}$$
   $$\sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3}$$
   Multiply them:
   $$5\sqrt{3} \times 2\sqrt{3} = 10 \times 3 = 30$$

Question A2.2 Find the values of x and y given that:
$$(3\sqrt{5} - 2\sqrt{3})^2 = x + y\sqrt{15}$$

Solution:
Expand the left-hand side using the perfect square identity $(a-b)^2 = a^2 - 2ab + b^2$:
$$(3\sqrt{5})^2 - 2(3\sqrt{5})(2\sqrt{3}) + (2\sqrt{3})^2$$
$$= (9 \times 5) - 12\sqrt{15} + (4 \times 3)$$
$$= 45 - 12\sqrt{15} + 12$$
$$= 57 - 12\sqrt{15}$$
Equating this to $x + y\sqrt{15}$ yields:

- $x = 57$
- $y = -12$

Question A2.3 Simplify by rationalising the denominator:

1. $\frac{10}{\sqrt{5}}$
   Solution:
   Multiply the numerator and denominator by $\sqrt{5}$:
   $$\frac{10 \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}} = \frac{10\sqrt{5}}{5} = 2\sqrt{5}$$

2. $\frac{1}{5-2\sqrt{3}}$
   Solution:
   Multiply the numerator and denominator by the conjugate $(5 + 2\sqrt{3})$:
   $$\frac{1(5 + 2\sqrt{3})}{(5 - 2\sqrt{3})(5 + 2\sqrt{3})} = \frac{5 + 2\sqrt{3}}{5^2 - (2\sqrt{3})^2} = \frac{5 + 2\sqrt{3}}{25 - 12} = \frac{5 + 2\sqrt{3}}{13}$$

3. $\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}} + \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$
   Solution:
   Find a common denominator by cross-multiplying the terms:
   $$\frac{(\sqrt{5} + \sqrt{2})^2 + (\sqrt{5} - \sqrt{2})^2}{(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2})}$$
   Expand the numerators and the difference of squares in the denominator:
   $$\text{Denominator} = (\sqrt{5})^2 - (\sqrt{2})^2 = 5 - 2 = 3$$
   $$\text{Numerator} = (5 + 2\sqrt{10} + 2) + (5 - 2\sqrt{10} + 2) = 7 + 7 = 14$$
   Combine them together:
   $$\frac{14}{3}$$

Question A2.4 Express the following in index form:

1. $\frac{4p^5}{\sqrt{y^3}}$
   Solution:
   Convert the radical to a fractional index: $\sqrt{y^3} = y^{3/2}$.
   $$\frac{4p^5}{y^{3/2}} = 4p^5y^{-3/2}$$

2. $5\sqrt{7}$
   Solution:
   Convert the square root to a fractional index:
   $$5 \cdot 7^{1/2}$$

Question A2.5 Expand and simplify $(\sqrt{5}-1)^4$. Show all working.

Solution:

We can calculate this by squaring $(\sqrt{5}-1)^2$ first, then squaring the result: 1. First, find the square:

$$(\sqrt{5}-1)^2 = (\sqrt{5})^2 - 2\sqrt{5} + 1 = 5 - 2\sqrt{5} + 1 = 6 - 2\sqrt{5}$$

2. Now, square that result:

$$\begin{aligned}(\sqrt{5}-1)^4 &= (6-2\sqrt{5})^2 \\ &= 6^2 - 2(6)(2\sqrt{5}) + (2\sqrt{5})^2 \\ &= 36 - 24\sqrt{5} + 20 \\ &= 56 - 24\sqrt{5}\end{aligned}$$

2 SECTION B: EQUATIONS, INEQUALITIES AND SIMULTANEOUS EQUATIONS

2.1 B1 - Linear Equations

Question B1.1 Solve each equation:

1. $7x - 4 = 3x + 12$

   Solution:
   Subtract $3x$ from both sides:
   $$4x - 4 = 12$$
   Add 4 to both sides:
   $$4x = 16$$
   Divide by 4:
   $$x = 4$$

2. $\frac{4x+3}{5} = 3$

   Solution:
   Multiply both sides by 5:
   $$4x + 3 = 15$$
   Subtract 3:
   $$4x = 12$$
   Divide by 4:
   $$x = 3$$

3. 5 times the sum of a number and 3 equals 3 more than the number. Find the number.

   Solution:
   Let the number be n. Write the algebraic phrase:
   $$5(n + 3) = n + 3$$
   Expand the left side:
   $$5n + 15 = n + 3$$
   Subtract n from both sides:
   $$4n + 15 = 3$$
   Subtract 15 from both sides:
   $$4n = -12$$
   Divide by 4:
   $$n = -3$$

Question B1.2 Solve the equation:

$$4^x \cdot 8^{-x} = 2^{3x-5}$$

Solution:
Convert all terms to base 2:
$$(2^2)^x \cdot (2^3)^{-x} = 2^{3x-5}$$
$$2^{2x} \cdot 2^{-3x} = 2^{3x-5}$$

$$2^{-x} = 2^{3x-5}$$

Equate the indices:

$$-x = 3x - 5$$

Add x to both sides:

$$0 = 4x - 5$$

$$4x = 5 \implies x = \frac{5}{4}$$

2.2 B2 - Inequalities

Question B2.1 Solve and graph each inequality on the number line provided:

1. $3x + 7 \geq -2$

   Solution:

   $$3x \geq -9 \implies x \geq -3$$

   Graphing reference: A solid/closed circle at -3 *with an arrow pointing to the right.*

2. $-5x < 20$

   Solution:
   Divide by -5 and flip the inequality sign:

   $$x > -4$$

   Graphing reference: An open circle at -4 *with an arrow pointing to the right.*

3. $2(x - 1) \leq x + 5$

   Solution:

   $$2x - 2 \leq x + 5$$

   $$x - 2 \leq 5 \implies x \leq 7$$

   Graphing reference: A solid/closed circle at 7 *with an arrow pointing to the left.*

Question B2.2 Solve the following:

1. Find all integers n satisfying $-3 < 2n - 1 \leq 9$.

   Solution:
   Add 1 to all parts of the inequality:

   $$-2 < 2n \leq 10$$

   Divide all parts by 2:

   $$-1 < n \leq 5$$

   The integers (n) that satisfy this range are:

   $$n = \{0, 1, 2, 3, 4, 5\}$$

2. A student needs at least 240 points across four equal tests to pass. They scored 55, 63, and 58 on the first three. What is the minimum score needed on the fourth test?

   Solution:
   Let the fourth test score be s:

$$55 + 63 + 58 + s \geq 240$$

$$176 + s \geq 240$$

$$s \geq 240 - 176 \implies s \geq 64$$

   The minimum score needed on the fourth test is **64**.

2.3 B3 - Simultaneous Equations

Question B3.1 Solve simultaneously using substitution or elimination:

1. $3x + y = 11$ and $x - 2y = 2$

   Solution:
   From the second equation, express x in terms of y:

$$x = 2y + 2$$

   Substitute this into the first equation:

$$3(2y + 2) + y = 11$$

$$6y + 6 + y = 11$$

$$7y + 6 = 11 \implies 7y = 5 \implies y = \frac{5}{7}$$

   Find x:

$$x = 2\left(\frac{5}{7}\right) + 2 = \frac{10}{7} + \frac{14}{7} = \frac{24}{7}$$

   Solution: $\left(\frac{24}{7}, \frac{5}{7}\right)$

2. $5x + 3y = 16$ and $2x - y = 1$

   Solution:
   From the second equation, express y:

$$y = 2x - 1$$

   Substitute into the first equation:

$$5x + 3(2x - 1) = 16$$

$$5x + 6x - 3 = 16$$

$$11x = 19 \implies x = \frac{19}{11}$$

   Find y:

$$y = 2\left(\frac{19}{11}\right) - 1 = \frac{38}{11} - \frac{11}{11} = \frac{27}{11}$$

   Solution: $\left(\frac{19}{11}, \frac{27}{11}\right)$

Question B3.2 Solve the following pairs of simultaneous equations:

1. $y = 2x - 3$ and $3x - 2y = 8$

   Solution:
   Substitute the first directly into the second equation:

   $$3x - 2(2x - 3) = 8$$

   $$3x - 4x + 6 = 8$$

   $$-x + 6 = 8 \implies -x = 2 \implies x = -2$$

   Find y:

   $$y = 2(-2) - 3 = -7$$

   Solution: $(-2, -7)$

2. $\frac{x}{2} - \frac{y}{3} = 1$ and $\frac{x}{3} + \frac{y}{2} = 2$

   Solution:
   Multiply the first equation by 6 to clear fractions:

   $$3x - 2y = 6 \quad \text{— (Eq 1)}$$

   Multiply the second equation by 6 to clear fractions:

   $$2x + 3y = 12 \quad \text{— (Eq 2)}$$

   Multiply (Eq 1) by 3 and (Eq 2) by 2:

   $$9x - 6y = 18$$

   $$4x + 6y = 24$$

   Add the two equations together to eliminate y:

   $$13x = 42 \implies x = \frac{42}{13}$$

   Substitute x back into (Eq 1) to solve for y:

   $$3\left(\frac{42}{13}\right) - 2y = 6$$

   $$\frac{126}{13} - 6 = 2y \implies \frac{126 - 78}{13} = 2y \implies \frac{48}{13} = 2y \implies y = \frac{24}{13}$$

   Solution: $\left(\frac{42}{13}, \frac{24}{13}\right)$

Question B3.3 A store sells pens at \$3 each and notebooks at \$5 each. Jake bought a total of 12 items and spent \$46. How many of each item did he buy? Set up and solve a pair of simultaneous equations.

Solution:
Let p be the number of pens and n be the number of notebooks.

$$\text{Item Equation: } p + n = 12 \implies p = 12 - n$$

$$\text{Cost Equation: } 3p + 5n = 46$$

Substitute the item expression into the cost equation:

$$3(12 - n) + 5n = 46$$

$$36 - 3n + 5n = 46$$

$$36 + 2n = 46$$

$$2n = 10 \implies n = 5$$

Find p:

$$p = 12 - 5 = 7$$

Jake bought **7 pens** and **5 notebooks**.

3 SECTION C: QUADRATICS

3.1 C1 – Solving Quadratic Equations

Question C1.1 Solve by factorising:

1. $x^2 7x + 12 = 0$

   Solution:
   Find numbers that multiply to 12 and add to -7 (-3 and -4):

   $$(x-3)(x-4) = 0 \implies x = 3 \text{ or } x = 4$$

2. $2x^2 7x 15 = 0$

   Solution:
   Find factors of $2 \times (-15) = -30$ that add to -7 (-10 and 3):

   $$2x^2 - 10x + 3x - 15 = 0$$

   $$2x(x-5) + 3(x-5) = 0$$

   $$(2x+3)(x-5) = 0 \implies x = -\frac{3}{2} \text{ or } x = 5$$

3. $5x^2 20 = 0$

   Solution:
   Factor out 5:

   $$5(x^2 - 4) = 0$$

   $$5(x-2)(x+2) = 0 \implies x = 2 \text{ or } x = -2$$

Question C1.2 Solve by completing the square (leave answers as exact values):

1. $x^2 + 8x 5 = 0$

   Solution:
   Move the constant: $x^2 + 8x = 5$
   Add $\left(\frac{8}{2}\right)^2 = 16$ to both sides:

   $$x^2 + 8x + 16 = 5 + 16$$

   $$(x+4)^2 = 21$$

   $$x + 4 = \pm\sqrt{21} \implies x = -4 \pm \sqrt{21}$$

2. $3x^2 + 12x 9 = 0$

   Solution:
   Divide by 3: $x^2 + 4x - 3 = 0 \implies x^2 + 4x = 3$
   Add $\left(\frac{4}{2}\right)^2 = 4$ to both sides:

   $$x^2 + 4x + 4 = 3 + 4$$

   $$(x+2)^2 = 7$$

   $$x + 2 = \pm\sqrt{7} \implies x = -2 \pm \sqrt{7}$$

Question C1.3 Use the quadratic formula. Leave answers as exact values:

1. $3x^25x7 = 0$

   Solution:
   Using $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$ where $a = 3, b = -5, c = -7$:

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(-7)}}{2(3)}$$

$$x = \frac{5 \pm \sqrt{25 + 84}}{6} = \frac{5 \pm \sqrt{109}}{6}$$

2. $5x^2 = 9x2$

   Solution:
   Rearrange to standard form: $5x^2 - 9x + 2 = 0$ $(a = 5, b = -9, c = 2)$:

$$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(5)(2)}}{2(5)}$$

$$x = \frac{9 \pm \sqrt{81 - 40}}{10} = \frac{9 \pm \sqrt{41}}{10}$$

Question C1.4 Harder solving:

1. Solve $3(x + 2)^24(x + 2)15 = 0$ by using the substitution $u = x + 2$.

   Solution:
   Rewrite using u:

$$3u^2 - 4u - 15 = 0$$

   Factorise: $(3u + 5)(u - 3) = 0 \implies u = -\frac{5}{3}$ or $u = 3$
   Substitute back $u = x + 2$:

   - $x + 2 = -\frac{5}{3} \implies x = -\frac{11}{3}$
   - $x + 2 = 3 \implies x = 1$

2. Solve $\frac{1}{x+2} + \frac{1}{x+3} = 1$

   Solution:
   Multiply all terms by the common denominator $(x + 2)(x + 3)$:

$$(x + 3) + (x + 2) = (x + 2)(x + 3)$$

$$2x + 5 = x^2 + 5x + 6$$

   Rearrange into standard quadratic form:

$$x^2 + 3x + 1 = 0$$

   Apply the quadratic formula $(a = 1, b = 3, c = 1)$:

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(1)}}{2(1)} = \frac{-3 \pm \sqrt{5}}{2}$$

3.2 C2 – The Discriminant

Question C2.1

1. Show that $x^2 + 6x + 14 = 0$ has no real roots.

   Solution:
   Calculate the discriminant $\Delta = b^2 - 4ac$:

   $$\Delta = 6^2 - 4(1)(14) = 36 - 56 = -20$$

   Since $\Delta < 0$, the quadratic equation has no real roots.

2. Find the value(s) of k for which $kx^2 4x + 1 = 0$ has exactly one solution.

   Solution:
   For exactly one solution, set $\Delta = 0$:

   $$\Delta = (-4)^2 - 4(k)(1) = 0$$

   $$16 - 4k = 0 \implies 4k = 16 \implies k = 4$$

3. Determine the number of real solutions of $35x^2 3x^4 + 920 = 0$. Justify your answer.

   Solution:
   Let $u = x^2$. This yields standard quadratic form: $-3u^2 + 35u + 920 = 0$.
   Find discriminant regarding u:

   $$\Delta_u = 35^2 - 4(-3)(920) = 1225 + 11040 = 12265 > 0$$

   This yields two real values for u:

   $$u = \frac{-35 \pm \sqrt{12265}}{-6}$$

   Since $\sqrt{12265} \approx 110.75$, one value of u is negative and one value of u is positive.

   - Because $u = x^2$, the negative u value yields no real solutions for x.
   - The positive u value yields two distinct real solutions ($x = \pm\sqrt{u}$).

   Thus, the original equation has **2 real solutions**.

Question C2.2 For the quadratic $y = 2x^2 kx + 8$, find the values of k for which the parabola has no x-intercepts.

Solution:

No x-intercepts means $\Delta < 0$:

$$\Delta = (-k)^2 - 4(2)(8) < 0$$

$$k^2 - 64 < 0$$

$$(k - 8)(k + 8) < 0 \implies -8 < k < 8$$

3.3 C3 – Parabola Features and Sketching

Question C3.1 For each parabola, find the vertex:

1. $y = 3x^2 12x + 7$ (by completing the square)

   Solution:

   $$y = 3(x^2 - 4x) + 7$$

   Add and subtract inside the bracket:

   $$y = 3(x^2 - 4x + 4 - 4) + 7$$
   $$y = 3((x-2)^2 - 4) + 7 = 3(x-2)^2 - 12 + 7$$
   $$y = 3(x-2)^2 - 5$$

   The vertex is $(2, -5)$.

2. $y = (3x2)(x + 5)$

   Solution:
   Find the x-intercepts by setting $y = 0$: $x = \frac{2}{3}$ and $x = -5$.
   The axis of symmetry lies halfway between them:

   $$x_v = \frac{\frac{2}{3} + (-5)}{2} = \frac{-\frac{13}{3}}{2} = -\frac{13}{6}$$

   Find the y-value of the vertex:

   $$y_v = \left(3\left(-\frac{13}{6}\right) - 2\right)\left(-\frac{13}{6} + 5\right) = \left(-\frac{13}{2} - \frac{4}{2}\right)\left(\frac{17}{6}\right) = \left(-\frac{17}{2}\right)\left(\frac{17}{6}\right) = -\frac{289}{12}$$

   The vertex is $\left(-\frac{13}{6}, -\frac{289}{12}\right)$.

Question C3.2 For the parabola $y = 2x^2 + 4x + 6$:

1. Express the equation in vertex form by completing the square.

   Solution:

   $$y = -2(x^2 - 2x) + 6$$
   $$y = -2(x^2 - 2x + 1 - 1) + 6$$
   $$y = -2(x-1)^2 + 2 + 6 \implies y = -2(x-1)^2 + 8$$

2. State the vertex and axis of symmetry.

   Solution:
   Vertex: $(1, 8)$, Axis of Symmetry: $x = 1$

3. Find the x- and y-intercepts.

   Solution:
   y-intercept (set $x = 0$): $y = 6 \implies \mathbf{(0, 6)}$
   x-intercepts (set $y = 0$):

   $$-2(x-1)^2 + 8 = 0 \implies 2(x-1)^2 = 8 \implies (x-1)^2 = 4$$
   $$x - 1 = \pm 2 \implies x = 3 \text{ or } x = -1 \implies \mathbf{(3, 0)}, \mathbf{(-1, 0)}$$

4. Sketch the parabola, showing all important features.

   Solution:
   Plot a downward-opening parabola peaking at $(1, 8)$, crossing the y-axis at 6, and passing through horizontal axis intercepts $(-1, 0)$ and $(3, 0)$.

3.4 C4 – Applications of Quadratics

Question C4.1 A ball is launched vertically upward. Its height h metres after t seconds is given by $h = 30t5t^2$.

1. At what time(s) is the ball 40 metres above the ground?

   Solution:

   $$40 = 30t - 5t^2 \implies 5t^2 - 30t + 40 = 0$$

   Divide by 5:

   $$t^2 - 6t + 8 = 0 \implies (t-2)(t-4) = 0$$

   The ball is at 40 metres at $t = 2$ **seconds** and $t = 4$ **seconds**.

2. After how many seconds does the ball hit the ground?

   Solution:
   Set $h = 0$:

   $$30t - 5t^2 = 0 \implies 5t(6-t) = 0$$

   Since $t > 0$ for the landing, $t = 6$ **seconds**.

3. Can the ball reach a height of 50 metres? Justify using the discriminant.

   Solution:
   Set $h = 50$:

   $$50 = 30t - 5t^2 \implies 5t^2 - 30t + 50 = 0 \implies t^2 - 6t + 10 = 0$$

   Find discriminant:

   $$\Delta = (-6)^2 - 4(1)(10) = 36 - 40 = -4$$

   Since $\Delta < 0$, there are no real solutions. **No**, the ball cannot reach 50 metres.

Question C4.2 The combined area of a rectangle and a square is 50 cm^2. The square has side length x cm. The rectangle has the same width x cm but its length is 3 cm longer than x. Write an equation in x and solve it to find x.

Solution:

$$\text{Area}_{\text{square}} = x^2$$
$$\text{Area}_{\text{rectangle}} = x(x+3) = x^2 + 3x$$
$$\text{Total Area} = x^2 + x^2 + 3x = 50 \implies 2x^2 + 3x - 50 = 0$$

Using the quadratic formula:

$$x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-50)}}{2(2)} = \frac{-3 \pm \sqrt{9 + 400}}{4} = \frac{-3 \pm \sqrt{409}}{4}$$

Since geometric dimension must be positive, pick the addition alternative:

$$x = \frac{-3 + \sqrt{409}}{4} \approx 4.31 \text{ cm}$$

4 SECTION D: LINEAR AND NON-LINEAR RELATIONSHIPS

4.1 D1 – Parabolas

Question D1.1

[Image of a quadratic parabola function graph]

[Image of a quadratic parabola function graph] For the parabola $y = (x3)(x + 1)$:

1. **State the x-intercepts and y-intercept.**

   Solution:
   x-intercepts (set $y = 0$): $x = 3$ and $x = -1$
   y-intercept (set $x = 0$): $y = (-3)(1) = -3$

2. **Find the vertex.**

   Solution:
   Axis of symmetry is $x = \frac{3+(-1)}{2} = 1$.
   Substitute $x = 1$: $y = (1 - 3)(1 + 1) = (-2)(2) = -4$.
   Vertex is $(1, -4)$.

3. **Sketch the parabola, labelling all key features.**

   Solution:
   Draw an upright parabola with its turning point at $(1, -4)$, cutting horizontal axes markers at -1 and 3, and hitting the vertical line boundary at -3.

Question D1.2 Describe the transformations applied to $y = x^2$ to obtain each of the following, then sketch:

1. **$y = (x + 2)^2 + 5$**

   Solution:
   Reflected across the x-axis, shifted horizontally left by 2 units, and shifted vertically up by 5 units. Vertex sits at $(-2, 5)$.

2. **$y = 3(x1)^2 4$**

   Solution:
   Vertically stretched by a factor of 3, shifted horizontally right by 1 unit, and shifted vertically down by 4 units. Vertex sits at $(1, -4)$.

4.2 D2 – Hyperbolas

Question D2.1 For the hyperbola $y = \frac{4}{x}$:

1. **State the domain and range.**

   Solution:
   Domain: All real x except $x \neq 0$, Range: All real y except $y \neq 0$

2. **State the equations of the asymptotes.**

   Solution:
   Vertical Asymptote: $x = 0$, Horizontal Asymptote: $y = 0$

3. **Sketch the graph, labelling key features.**

   Solution:
   Standard hyperbola branches situated cleanly within Quadrants I and III; sample coordinates include $(2, 2)$ and $(-2, -2)$.

Question D2.2 Describe how $y = -\frac{2}{x-3} + 1$ differs from $y = \frac{1}{x}$, and sketch it showing asymptotes and any intercepts.

1. **State the equations of the asymptotes.**

   Solution:
   Reflected across the x-axis, vertically scaled by **2**, shifted right **3** units, and up **1** unit.
   Vertical Asymptote: $x = 3$, Horizontal Asymptote: $y = 1$

2. **Find the x- and y-intercepts.**

   Solution:
   y-intercept $(x = 0)$: $y = -\frac{2}{0-3} + 1 = \frac{2}{3} + 1 = \mathbf{\frac{5}{3}}$
   x-intercept $(y = 0)$: $0 = -\frac{2}{x-3} + 1 \implies \frac{2}{x-3} = 1 \implies x - 3 = 2 \implies \mathbf{x = 5}$

3. **Sketch the graph.**

   Solution:
   Draw branches localized around shifting layout boundaries $x = 3$ and $y = 1$ crossing the axes at $(0, 5/3)$ and $(5, 0)$.

4.3 D3 – Exponential Functions

Question D3.1
[Image of an exponential growth function graph]
[Image of an exponential growth function graph] Consider the function $y = 3\times 2^x$:

1. **Complete the table of values:**

   Solution:

x	-2	-1	0	1	2
y	$\frac{3}{4}$	$\frac{3}{2}$	3	6	12

2. **Sketch the graph, labelling the y-intercept and at least one other point.**

   Solution:
   Plot an upward exponential curve rising left-to-right, displaying the crossing coordinate $(0, 3)$ clearly along with a key marker like $(1, 6)$.

3. **State the equation of the asymptote.**

   Solution:
   Horizontal Asymptote: $y = 0$

Question D3.2 Describe the transformations from $y = 2^x$ to $y = 2^{-x} + 4$ and sketch, showing the asymptote and y-intercept.

1. **Describe the transformations.**

   Solution:
   Reflected across both the x-axis and y-axis, and shifted vertically up by 4 units.

2. **Sketch the graph.**

Solution:
Asymptote: $y = 4$. **y-intercept** ($x = 0$): $y = -2^0 + 4 = -1 + 4 = 3 \implies (\mathbf{0}, \mathbf{3})$. **The curve approaches** $y = 4$ **towards the right and drops off downwards rapidly towards the left.**

4.4 D4 – Circles

Question D4.1

1. **Write the equation of a circle with centre** $(0, 0)$ **and radius 7.**

Solution:

$$x^2 + y^2 = 7^2 \implies x^2 + y^2 = 49$$

2. **Write the equation of a circle with centre** $(2, 5)$ **and radius 4.**

Solution:

$$(x - (-2))^2 + (y - 5)^2 = 4^2 \implies (x + 2)^2 + (y - 5)^2 = 16$$

3. **Determine whether the point** $A(3, 6)$ **lies on, inside, or outside the circle** $(x + 1)^2 + (y2)^2 = 20$. **Show working.**

Solution:
Substitute $(-3, 6)$ **directly into the equation:**

$$(-3 + 1)^2 + (6 - 2)^2 = (-2)^2 + (4)^2 = 4 + 16 = 20$$

Since the resulting evaluation matches 20 exactly, the coordinates point directly on the circle line layout.

Question D4.2 **For the circle** $(x2)^2 + (y + 3)^2 = 36$**:**

1. **State the centre and radius.**

Solution:
Centre: $(2, -3)$, **Radius: 6** $\quad (\sqrt{36})$

2. **Sketch the circle on the Cartesian plane.**

Solution:
Draw a circular layout with its center focused at $(2, -3)$ **tracking outward exactly 6 units in all perimeter directions.**

3. **Find the** y**-coordinates of the points on the circle where** $x = 6$.

Solution:
Substitute $x = 6$**:**

$$(6 - 2)^2 + (y + 3)^2 = 36$$
$$16 + (y + 3)^2 = 36$$
$$(y + 3)^2 = 20$$
$$y + 3 = \pm\sqrt{20} = \pm 2\sqrt{5} \implies \mathbf{y = -3 \pm 2\sqrt{5}}$$

4.5 D5 – Mixed Non-Linear (Challenge)

Question D5.1 Consider the parabola $y = (4x)(x+2)$ and the line $y = 6$.

1. **Find the coordinates of the vertex of the parabola.**

   Solution:
   Expand the parabola: $y = 4x + 8 - x^2 - 2x = -x^2 + 2x + 8$
   Axis of symmetry: $x = -\frac{b}{2a} = -\frac{2}{2(-1)} = 1$
   Substitute $x = 1$: $y = (4-1)(1+2) = 3 \times 3 = 9$
   Vertex is $(1, 9)$.

2. **Find the points of intersection of the parabola and the line.**

   Solution:
   Set equations equal:

   $$-x^2 + 2x + 8 = 6 \implies x^2 - 2x - 2 = 0$$

   Using the quadratic formula:

   $$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-2)}}{2(1)} = \frac{2 \pm \sqrt{12}}{2} = 1 \pm \sqrt{3}$$

   The coordinates of intersection are $(1+\sqrt{3}, 6)$ and $(1-\sqrt{3}, 6)$.

3. **Sketch both curves on the same set of axes, clearly indicating all important features and the region where $(4x)(x+2) \leq 6$.**

   Solution:
   The parabola loops downwards starting at vertex $(1, 9)$ crossing key axis marks at -2 and 4. The straight boundary line traces horizontally at $y = 6$.
   The inequality criteria $(4-x)(x+2) \leq 6$ targets where the parabola falls below or matches the line layout, returning intervals: $x \leq 1 - \sqrt{3}$ or $x \geq 1 + \sqrt{3}$.