

ADVANCED (2027)
Stage 6

norWEST

CHRISTIAN COLLEGE

STRENGTH OF PURPOSE

Functions (Adv-2027)
Working with Functions (Y11)
Algebraic Techniques
Teacher: Jana Thorrold
Exam Equivalent Time: 34.5 minutes *(based on allocation of 1.5 minutes per mark)*

Questions

1. Functions, 2ADV F1 2012 HSC 2 MC

Which of the following is equal to $\frac{1}{2\sqrt{5} - \sqrt{3}}$?

- A. $\frac{2\sqrt{5} - \sqrt{3}}{7}$
- B. $\frac{2\sqrt{5} + \sqrt{3}}{7}$
- C. $\frac{2\sqrt{5} - \sqrt{3}}{17}$
- D. $\frac{2\sqrt{5} + \sqrt{3}}{17}$

2. Functions, 2ADV F1 EQ-Bank 11

a. If $\frac{1}{\sqrt[3]{7} + \pi} = (7 + \pi)^x$, find x . (1 mark)

b. Calculate the value of $\frac{1}{\sqrt[3]{7} + \pi}$ to 3 significant figures. (1 mark)

3. Functions, 2ADV F1 2011 HSC 1f

Rationalise the denominator of $\frac{4}{\sqrt{5} - \sqrt{3}}$.

Give your answer in the simplest form. (2 marks)

4. Functions, 2ADV EQ-Bank 12

Rationalise the denominator in $\frac{\sqrt{3} - \sqrt{2}}{\sqrt{5} + \sqrt{2}}$, and express in the simplest form. (2 marks)

5. Functions, 2ADV F1 2008 HSC 1e

Expand and simplify $(\sqrt{3} - 1)(2\sqrt{3} + 5)$. (2 marks)

6. Functions, 2ADV F1 EQ-Bank 17

Find a and b such that a, b are real numbers and

$$\frac{8 - \sqrt{27}}{2\sqrt{3}} = a + b\sqrt{3}. \quad (2 \text{ marks})$$

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7. Functions, 2ADV F1 EQ-Bank 18

Find a and b such that a, b are real numbers and

$$\frac{\sqrt{3} - 2}{2\sqrt{3}} = a + b\sqrt{3}. \quad (2 \text{ marks})$$

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8. Functions, 2ADV F1 EQ-Bank 19

Find a and b such that a, b are real numbers and

$$\frac{6\sqrt{3} - \sqrt{5}}{2\sqrt{5}} = a + b\sqrt{15}. \quad (2 \text{ marks})$$

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9. Functions, 2ADV F1 2004 HSC 1d

Find integers a and b by showing working to expand and simplify

$$\left(3 - \sqrt{2}\right)^2 = a - b\sqrt{2}. \quad (2 \text{ marks})$$

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10. Functions, 2ADV F1 EQ Bank 25

Find a and b such that a and b are real and $\frac{2\sqrt{3} + 2}{\sqrt{6} - \sqrt{2}} = a\sqrt{2} + b\sqrt{6}$. (2 marks)

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11. Functions, 2ADV EQ-Bank 27

Simplify $(27x^6)^{\frac{1}{3}} \times (2x)^{-2}$. (2 marks)

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12. Functions, 2ADV F1 EQ-Bank 31

Simplify $\left(\frac{p}{q}\right)^3 \div (pq^{-2})$. (2 marks)

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Worked Solutions

1. Functions, 2ADV F1 2012 HSC 2 MC

$$\begin{aligned}\frac{1}{2\sqrt{5} - \sqrt{3}} &\times \frac{2\sqrt{5} + \sqrt{3}}{2\sqrt{5} + \sqrt{3}} \\&= \frac{2\sqrt{5} + \sqrt{3}}{(2\sqrt{5} - \sqrt{3})(2\sqrt{5} + \sqrt{3})} \\&= \frac{2\sqrt{5} + \sqrt{3}}{(2\sqrt{5})^2 - (\sqrt{3})^2} \\&= \frac{2\sqrt{5} + \sqrt{3}}{17} \\&\Rightarrow D\end{aligned}$$

2. Functions, 2ADV F1 EQ-Bank 11

$$\begin{aligned}\text{a. } \frac{1}{\sqrt[3]{7} + \pi} &= (7 + \pi)^{-\frac{1}{3}} \\ \text{b. } \frac{1}{\sqrt[3]{7} + \pi} &= 0.4619\dots \\ &= 0.462 \quad (3 \text{ sig. fig.})\end{aligned}$$

3. Functions, 2ADV F1 2011 HSC 1f

$$\begin{aligned}\frac{4}{\sqrt{5} - \sqrt{3}} &\times \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} + \sqrt{3}} \\&= \frac{4(\sqrt{5} + \sqrt{3})}{5 - 3} \\&= 2(\sqrt{5} + \sqrt{3})\end{aligned}$$

4. Functions, 2ADV EQ-Bank 12

$$\begin{aligned}\frac{\sqrt{3} - \sqrt{2}}{\sqrt{5} + \sqrt{2}} &\times \frac{\sqrt{5} - \sqrt{2}}{\sqrt{5} - \sqrt{2}} \\&= \frac{(\sqrt{3} - \sqrt{2})(\sqrt{5} - \sqrt{2})}{5 - 2} \\&= \frac{\sqrt{15} - \sqrt{6} - \sqrt{10} + 2}{3}\end{aligned}$$

5. Functions, 2ADV F1 2008 HSC 1e

$$\begin{aligned}(\sqrt{3} - 1)(2\sqrt{3} + 5) \\&= 2 \times 3 + 5\sqrt{3} - 2\sqrt{3} - 5 \\&= 1 + 3\sqrt{3}\end{aligned}$$

6. Functions, 2ADV F1 EQ-Bank 17

$$\begin{aligned}\frac{8 - \sqrt{27}}{2\sqrt{3}} \times \frac{2\sqrt{3}}{2\sqrt{3}} &= \frac{2\sqrt{3}(8 - 3\sqrt{3})}{(2\sqrt{3})^2} \\&= \frac{16\sqrt{3} - 18}{12} \\&= -\frac{3}{2} + \frac{4}{3}\sqrt{3} \\ \therefore a &= -\frac{3}{2}, \quad b = \frac{4}{3}\end{aligned}$$

7. Functions, 2ADV F1 EQ-Bank 18

$$\begin{aligned}\frac{\sqrt{3} - 2}{2\sqrt{3}} &= \frac{\sqrt{3} - 2}{2\sqrt{3}} \times \frac{2\sqrt{3}}{2\sqrt{3}} \\&= \frac{2\sqrt{3}(\sqrt{3} - 2)}{4 \times 3} \\&= \frac{6 - 4\sqrt{3}}{12} \\&= \frac{1}{2} - \frac{1}{3}\sqrt{3} \\ \therefore a &= \frac{1}{2}, \quad b = -\frac{1}{3}\end{aligned}$$

8. Functions, 2ADV F1 EQ-Bank 19

$$\begin{aligned}\frac{6\sqrt{3} - \sqrt{5}}{2\sqrt{5}} &= \frac{6\sqrt{3} - \sqrt{5}}{2\sqrt{5}} \times \frac{2\sqrt{5}}{2\sqrt{5}} \\ &= \frac{2\sqrt{5}(6\sqrt{3} - \sqrt{5})}{4 \times 5} \\ &= \frac{12\sqrt{15} - 10}{20} \\ &= -\frac{1}{2} + \frac{3}{5}\sqrt{15}\end{aligned}$$

$$\therefore a = -\frac{1}{2}, b = \frac{3}{5}$$

9. Functions, 2ADV F1 2004 HSC 1d

$$\begin{aligned}(3 - \sqrt{2})^2 &= 9 - 6\sqrt{2} + (\sqrt{2})^2 \\ &= 9 - 6\sqrt{2} + 2 \\ &= 11 - 6\sqrt{2}\end{aligned}$$

$$\therefore a = 11, b = 6$$

10. Functions, 2ADV F1 EQ Bank 25

$$\begin{aligned}\frac{2\sqrt{3} + 2}{\sqrt{6} - \sqrt{2}} \times \frac{\sqrt{6} + \sqrt{2}}{\sqrt{6} + \sqrt{2}} \\ &= \frac{(2\sqrt{3} + 2)(\sqrt{6} + \sqrt{2})}{6 - 2} \\ &= \frac{2\sqrt{18} + 2\sqrt{6} + 2\sqrt{6} + 2\sqrt{2}}{4} \\ &= \frac{6\sqrt{2} + 4\sqrt{6} + 2\sqrt{2}}{4} \\ &= \frac{8\sqrt{2} + 4\sqrt{6}}{4} \\ &= 2\sqrt{2} + \sqrt{6}\end{aligned}$$

$$\therefore a = 2 \text{ and } b = 1$$

11. Functions, 2ADV EQ-Bank 27

$$\begin{aligned}(27x^6)^{\frac{1}{3}} \times (2x)^{-2} &= 27^{\frac{1}{3}} \times x^{6 \times \frac{1}{3}} \times \frac{1}{4x^2} \\ &= 3x^2 \times \frac{1}{4x^2} \\ &= \frac{3}{4}\end{aligned}$$

12. Functions, 2ADV F1 EQ-Bank 31

$$\begin{aligned}\left(\frac{p}{q}\right)^3 \div (pq^{-2}) &= \frac{p^3}{q^3} \div \frac{p}{q^2} \\ &= \frac{p^3}{q^3} \times \frac{q^2}{p} \\ &= \frac{p^2}{q}\end{aligned}$$