NWCC

Graphical Transformations Practice Set

Linear, Non-Linear Relationship Models & Intersections

PRACTICE PROBLEMS [1PT]

Question 1 **(4 marks)**

The graph of the line $y = 2x - 3$ is translated 4 units horizontally to the left, and then reflected across the x-axis.

(a) Find the equation of the resulting line in gradient-intercept form ($y = mx + c$). **(2 marks)**

(b) Sketch both the original line $y = 2x - 3$ and the transformed line on a single set of axes, labeling all axial intercepts clearly. **(2 marks)**

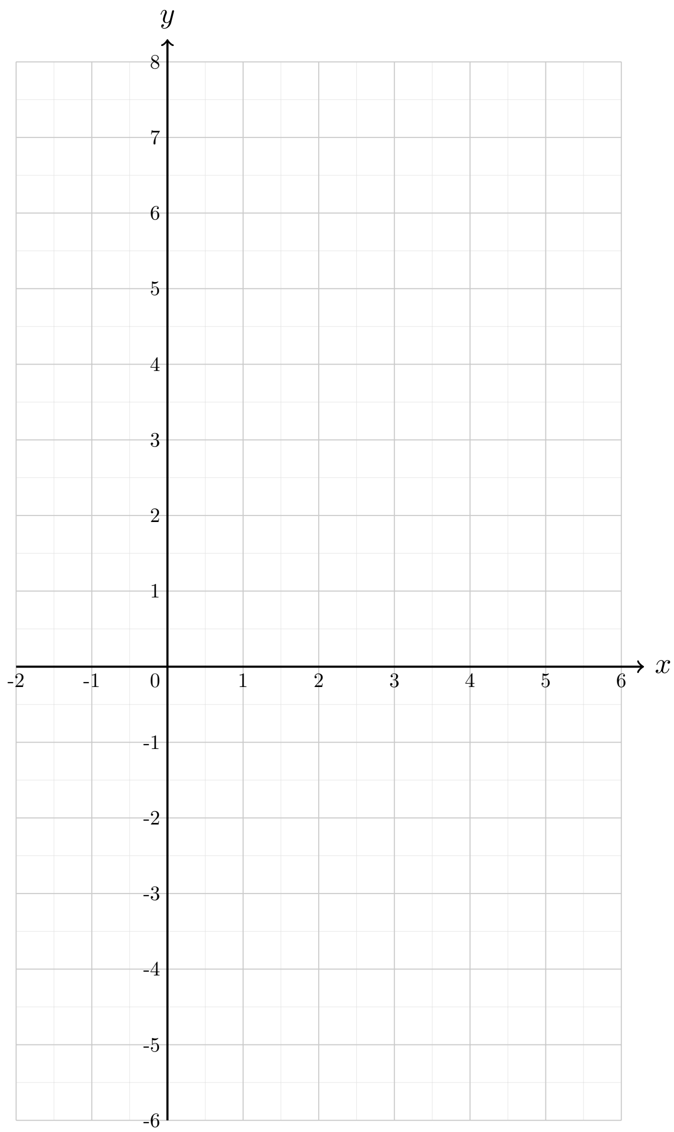
Question 2 **(5 marks)**

A parent parabola $y = x^2$ undergoes the following sequence of transformations:

- A vertical stretch by a factor of 3.
- A horizontal translation of 2 units to the right.
- A vertical translation of 5 units downwards.

(a) Write down the equation of the transformed parabola in vertex form, and state the coordinates of its new turning point. **(3 marks)**

(b) Sketch the transformed parabola on the grid below, explicitly labeling the vertex and the y-intercept. **(2 marks)**

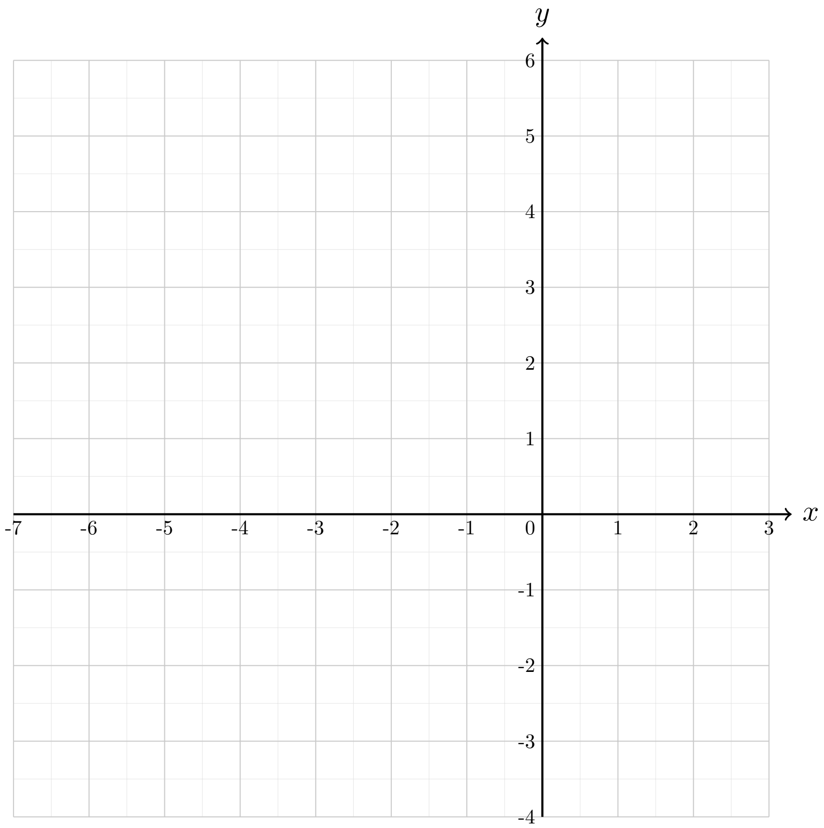


Question 3 **(5 marks)**

The basic hyperbola $y = \frac{1}{x}$ is transformed into the curve $y = \frac{2}{x+3} + 1$.

(a) Describe the three geometric transformations that have occurred and state the equations of both the vertical and horizontal asymptotes. **(3 marks)**

(b) Sketch this hyperbola on the grid below, showing both asymptotes as dashed lines and labeling the y-intercept. **(2 marks)**

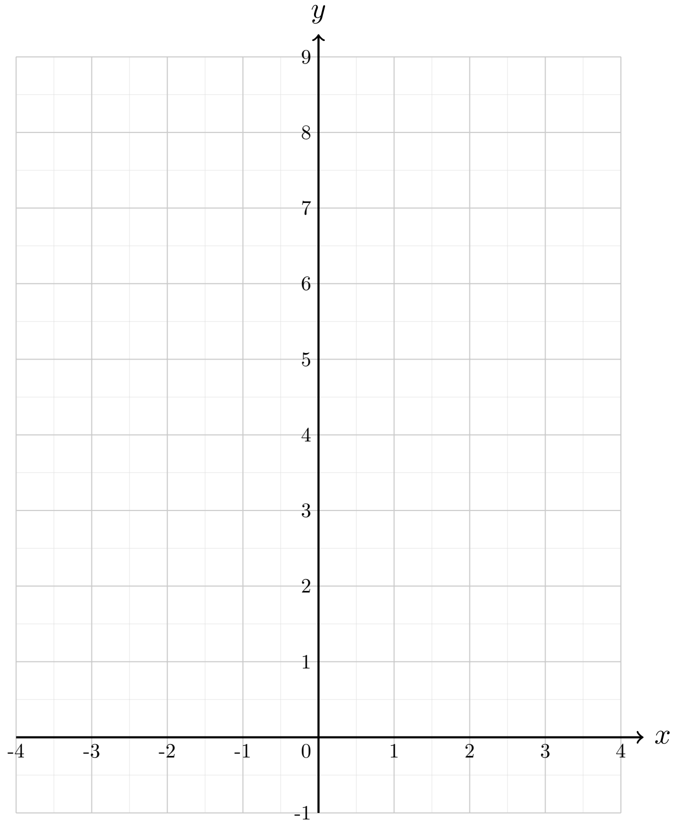


Question 4 **(5 marks)**

Consider the exponential curve $y = 3^x$. This graph is reflected across the y-axis, horizontally compressed by a factor of 2, and then shifted vertically upwards by 4 units.

(a) State the equation of the resulting curve and determine its new y-intercept. **(3 marks)**

(b) Sketch the transformed exponential curve on the grid below, labeling the horizontal asymptote and the y-intercept. **(2 marks)**

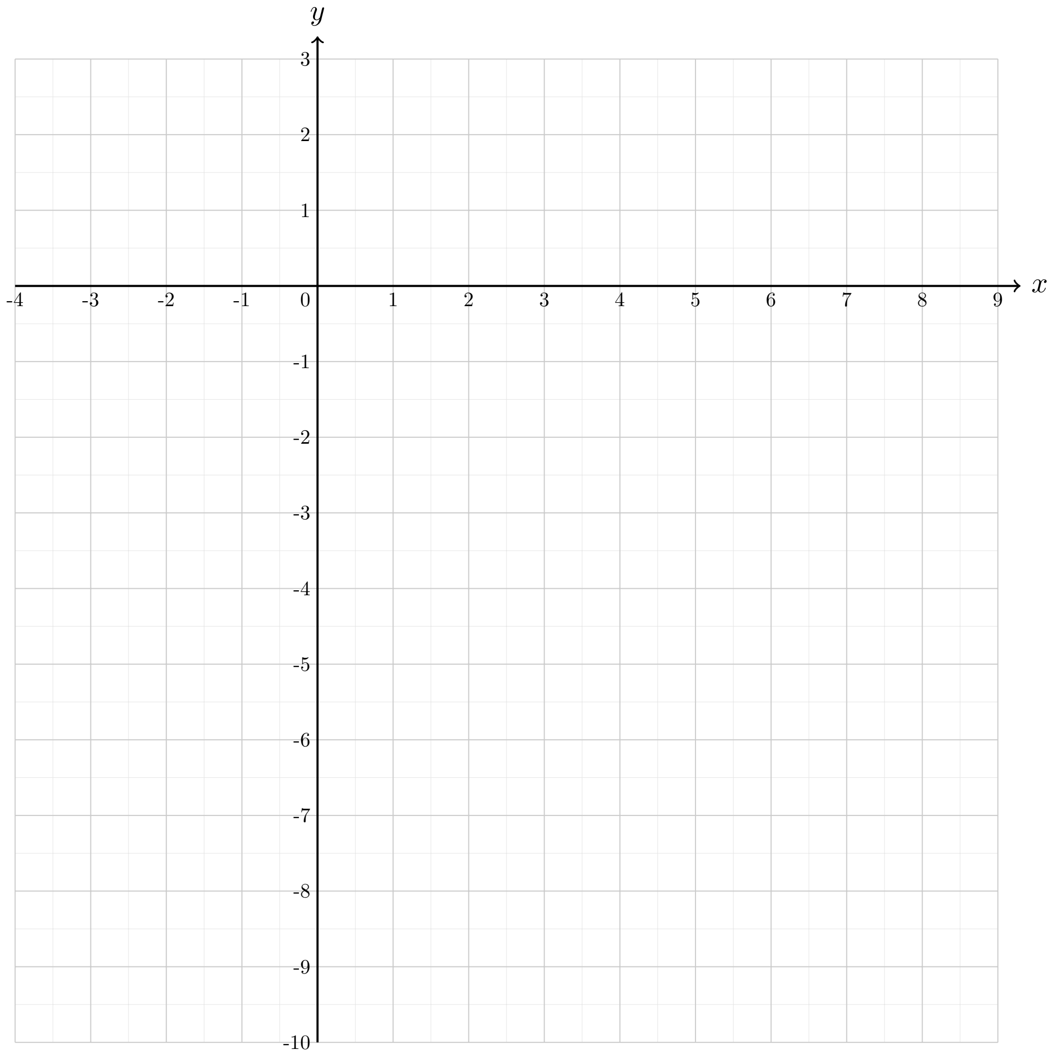


Question 5 **(5 marks)**

The circle centered at the origin with a radius of 5 has the equation $x^2 + y^2 = 25$. This circle is translated 3 units to the right and 4 units downwards.

(a) State the new equation of the circle and determine whether the origin $(0, 0)$ lies inside, outside, or exactly on the boundary of the transformed circle. **(3 marks)**

(b) Sketch the transformed circle on the grid below, clearly labeling its new center point and its intercepts with the coordinate axes. **(2 marks)**



Question 6 **(4 marks)**

A parabola with the equation $y = x^2$ is translated 2 units to the right and 1 unit upwards to form Curve A. A basic hyperbola $y = \dfrac{1}{x}$ is translated 2 units to the right to form Curve B.

(a) Write down the equations for both Curve A and Curve B.

(b) Show algebraically that Curve A and Curve B intersect at exactly one point, and find the coordinates of this point of intersection.

SOLUTIONS AND ANSWERS [1PT]

Question 1 Solution

✓ **(a) Equation setup:** Shifting 4 units left replaces x with $(x+4)$:
$y = 2(x+4) - 3 \implies y = 2x + 8 - 3 \implies y = 2x + 5$
Reflecting across x-axis negates the entire function: $y = -(2x+5) \implies \mathbf{y = -2x - 5}$

✓ **(b) Sketch features:** Original line $y = 2x - 3$ has a y-intercept at $(0, -3)$ and an x-intercept at $(1.5, 0)$.
Transformed line $y = -2x - 5$ has a y-intercept at $(0, -5)$ and an x-intercept at $(-2.5, 0)$.

Question 2 Solution

✓ **(a) Equation & Vertex:** Following transformations sequentially yields: $\mathbf{y = 3(x-2)^2 - 5}$.
The new turning point (vertex) shifts to $(\mathbf{2}, \mathbf{-5})$.

✓ **(b) Sketch features:** An upright parabola with vertex at $(2, -5)$. To find the y-intercept, substitute $x = 0$: $y = 3(0-2)^2 - 5 = 12 - 5 = 7$. Intercept occurs at $(\mathbf{0}, \mathbf{7})$.

Question 3 Solution

✓ **(a) Transformations & Asymptotes:** Transformations: Vertical stretch by scale factor 2, shift 3 units left, and shift 1 unit up.
Asymptotes shift along with the translations: $\mathbf{x = -3}$ and $\mathbf{y = 1}$.

✓ **(b) Sketch features:** Draw dashed lines for asymptotes at $x = -3$ and $y = 1$. The y-intercept occurs at $x = 0 \implies y = \frac{2}{0+3} + 1 = \frac{5}{3} \approx 1.67$, giving the point $(\mathbf{0}, \mathbf{1.67})$. Branches occupy the top-right and bottom-left regions relative to the new asymptotes.

Question 4 Solution

✓ **(a) Equation & Intercept:** Applying reflections, horizontal compressions, and vertical shifts yields: $\mathbf{y = 3^{-2x} + 4}$.
The y-intercept occurs where $x = 0 \implies y = 3^0 + 4 = 5$, giving coordinate $(\mathbf{0}, \mathbf{5})$.

✓ **(b) Sketch features:** A decaying exponential curve flattening out overhead a horizontal asymptote line sketched at $\mathbf{y = 4}$. It passes directly through the calculated y-intercept at $(0, 5)$.

Question 5 Solution

✓ **(a) Equation & Verification:** Moving the center to $(3, -4)$ gives the equation: $\mathbf{(x-3)^2 + (y+4)^2 = 25}$.
Substituting $(0, 0)$ gives $(0-3)^2 + (0+4)^2 = 9 + 16 = 25$. Because this perfectly matches the radius squared ($r^2 = 25$), the origin $(0, 0)$ lies **exactly on the boundary of the circle**.

✓ **(b) Sketch features:** A circle with center plotted at $(3, -4)$ and radius 5. Since $(0, 0)$ is a boundary point, the circle passes cleanly through the origin. Additional boundary tracking includes $(3, 1)$, $(3, -9)$, $(8, -4)$, and $(-2, -4)$.

Question 6 Solution

✓ **(a) Equations:**

$$\text{Curve A: } \mathbf{y = (x-2)^2 + 1}, \quad \text{Curve B: } \mathbf{y = \frac{1}{x-2}}$$

✓ **(b) Point of Intersection:** Equate both curves to isolate key root nodes:

$$(x-2)^2+1=\frac{1}{x-2} \implies (x-2)^3+(x-2)=1$$

Let $u=x-2$, then $u^3+u-1=0$. Inspecting for clear intersections yields $u=1 \implies x-2=1 \implies x=3$.
Substitute $x=3$ back into Curve B: $y=\frac{1}{3-2}=1$.
The unique intersection coordinate is verified at $(\mathbf{3},\mathbf{1})$.

— End of Transformations Problem Set —