ADVANCED (2027)

Stage 6

Functions (Adv-2027)
Working with Functions (Y11)
Algebraic Techniques

Teacher: Jana Thorrold

Exam Equivalent Time: 58.5 minutes *(based on allocation of 1.5 minutes per mark)*

Questions

1. Functions, 2ADV EQ-Bank 3 MC

The set of values of k for which $x^2 + 2x - k = 0$ has two real solutions is

A. $[-1, 1]$

B. $(-1, \infty)$

C. $(-\infty, -1)$

D. $[-1]$

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2. Functions, 2ADV EQ-Bank 4 MC

The graph of $y = kx - 3$ intersects the graph of $y = x^2 + 8x$ at two distinct points for

A. $k = 11$

B. $k > 8 + 2\sqrt{3}$ or $k < 8 - 2\sqrt{3}$

C. $5 \leq k \leq 6$

D. $8 - 2\sqrt{3} \leq k \leq 8 + 2\sqrt{3}$

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3. Functions, 2ADV F1 2004 HSC 1c

Solve $\frac{x-5}{3} - \frac{x+1}{4} = 5$. *(2 marks)*

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4. Functions, 2ADV F1 2005 HSC 1d

Express $\frac{(2x-3)}{2} - \frac{(x-1)}{5}$ as a single fraction in its simplest form. *(2 marks)*

5. Functions, 2ADV EQ-Bank 13

Simplify $\dfrac{x^2}{x^2-2x-15}-\dfrac{x}{x+3}$. *(2 marks)*

6. Functions, 2ADV EQ-Bank 14

Simplify $8ab^2\times 2a^{-2}$. *(2 marks)*

7. Functions, 2ADV EQ-Bank 16

Simplify $\dfrac{a^3b-ab^3}{a^2+2ab+b^2}$. *(2 marks)*

8. Functions, 2ADV F1 2021 HSC 11

Solve $x+\dfrac{x-1}{2}=9$. *(2 marks)*

9. Functions, 2ADV F1 EQ-Bank 22

Simplify $\dfrac{9x^2}{x+3} \div \dfrac{3x}{x^2-9}$. *(2 marks)*

10. Functions, 2ADV F1 EQ-Bank 20

Simplify $\dfrac{4p-12p^2}{3} \times \dfrac{6p}{3p^2-p}$. *(2 marks)*

11. Functions, 2ADV 2004 HSC 2c

For what values of k does $x^2 - kx + 4 = 0$ have no real roots? *(2 marks)*

12. Functions, 2ADV F1 2008 HSC 1c

Simplify $\dfrac{2}{n} - \dfrac{1}{n+1}$. *(2 marks)*

13. Functions, 2ADV F1 EQ-Bank 26

Find the reciprocal of $\frac{1}{a}+\frac{1}{b}-\frac{c}{ab}$. *(2 marks)*

14. Functions, 2ADV F1 EQ-Bank 29

Fully simplify the expression $\frac{4}{x^2-9}-\frac{2x+1}{x+3}$ *(3 marks)*

15. Functions, 2ADV F1 EQ-Bank 30

Find the value of k if $4kx^2-(3-4k)x+k=0$ has one root. *(2 marks)*

16. Functions, 2ADV F1 EQ-Bank 32

Show that the parabola $2x^2-kx+k-2$ has at least one real root. *(3 marks)*

17. Functions, 2ADV 2009 HSC 4b

Find the values of k for which the quadratic equation

$$x^2 - (k+4)x + (k+7) = 0$$

has equal roots. ***(3 marks)***

18. Functions, 2ADV 2015 HSC 12d

For what values of k does the quadratic equation $x^2 - 8x + k = 0$ have real roots? ***(2 marks)***

19. Functions, 2ADV EQ-Bank 37

Simplify $\dfrac{p+1}{q-q^3} \div \dfrac{p^3+p^2}{q^2-q}$. ***(2 marks)***

Worked Solutions

1. Functions, 2ADV EQ-Bank 3 MC

Two real solutions:

$$\begin{aligned} b^2 - 4ac &> 0 \\ 4 - 4 \cdot 1 \cdot (-k) &> 0 \\ 4k &> -4 \\ k &> -1 \end{aligned}$$

$k \in (-1, \infty)$

$\Rightarrow \quad B$

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2. Functions, 2ADV EQ-Bank 4 MC

Intersection occurs when:

$$\begin{aligned} kx - 3 &= x^2 + 8x \\ x^2 + (8-k)x + 3 &= 0 \end{aligned}$$

For 2 points of intersection:

$$\begin{aligned} \Delta &> 0 \\ (8-k)^2 - 4(3) &> 0 \\ (8-k)^2 &> 12 \end{aligned}$$

$\therefore k < 8 - 2\sqrt{3} \quad \cup \quad k > 8 + 2\sqrt{3}$

$\Rightarrow \quad B$

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3. Functions, 2ADV F1 2004 HSC 1c

$$\begin{aligned} \frac{x-5}{3} - \frac{x+1}{4} &= 5 \\ 12\left(\frac{x-5}{3}\right) - 12\left(\frac{x+1}{4}\right) &= 12 \times 5 \\ 4x - 20 - 3x - 3 &= 60 \\ x - 23 &= 60 \\ \therefore x &= 83 \end{aligned}$$

Worked Solutions

4. Functions, 2ADV F1 2005 HSC 1d

$$\begin{aligned} &\frac{(2x-3)}{2} - \frac{(x-1)}{5} \\ &= \frac{5(2x-3) - 2(x-1)}{10} \\ &= \frac{10x - 15 - 2x + 2}{10} \\ &= \frac{8x - 13}{10} \end{aligned}$$

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5. Functions, 2ADV EQ-Bank 13

$$\begin{aligned} \frac{x^2}{x^2 - 2x - 15} - \frac{x}{x+3} &= \frac{x^2}{(x+3)(x-5)} - \frac{x}{x+3} \\ &= \frac{x^2 - x(x-5)}{(x+3)(x-5)} \\ &= \frac{x^2 - x^2 + 5x}{(x+3)(x-5)} \\ &= \frac{5x}{(x+3)(x-5)} \end{aligned}$$

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6. Functions, 2ADV EQ-Bank 14

$$8ab^2 \times 2a^{-2} = 8ab^2 \times \frac{2}{a^2} = \frac{16b^2}{a}$$

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7. Functions, 2ADV EQ-Bank 16

$$\begin{aligned} \frac{a^3b - ab^3}{a^2 + 2ab + b^2} &= \frac{ab\left(a^2 - b^2\right)}{(a+b)^2} \\ &= \frac{ab(a+b)(a-b)}{(a+b)^2} \\ &= \frac{ab(a-b)}{a+b} \end{aligned}$$

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8. Functions, 2ADV F1 2021 HSC 11

$$\begin{aligned} x + \frac{x-1}{2} &= 9 \\ 2x + x - 1 &= 18 \\ 3x &= 19 \\ x &= \frac{19}{3} \end{aligned}$$

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9. Functions, 2ADV F1 EQ-Bank 22

$$\begin{aligned}\frac{9x^2}{x+3} \div \frac{3x}{x^2-9} &= \frac{9x^2}{x+3} \times \frac{x^2-9}{3x} \\ &= \frac{9x^2}{x+3} \times \frac{(x-3)(x+3)}{3x} \\ &= 3x(x-3)\end{aligned}$$

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10. Functions, 2ADV F1 EQ-Bank 20

$$\begin{aligned}\frac{4p-12p^2}{3} \times \frac{6p}{3p^2-p} &= \frac{4p(1-3p)}{3} \times \frac{6p}{p(3p-1)} \\ &= \frac{8p(1-3p)}{3p-1} \\ &= -8p\end{aligned}$$

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11. Functions, 2ADV 2004 HSC 2c

$x^2 - kx + 4 = 0$

No real roots when $\Delta < 0$

$$\begin{aligned}b^2 - 4ac &< 0 \\ (-k^2) - 4 \times 1 \times 4 &< 0 \\ k^2 - 16 &< 0 \\ k^2 &< 16 \\ \therefore \quad -4 < k &< 4\end{aligned}$$

$\therefore$ There are no real roots when $-4 < k < 4$.

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12. Functions, 2ADV F1 2008 HSC 1c

$$\begin{aligned}&\frac{2}{n} - \frac{1}{n+1} \\ &= \frac{2(n+1) - 1(n)}{n(n+1)} \\ &= \frac{2n+2-n}{n(n+1)} \\ &= \frac{n+2}{n(n+1)}\end{aligned}$$

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13. Functions, 2ADV F1 EQ-Bank 26

$$\begin{aligned}\frac{1}{a} + \frac{1}{b} - \frac{c}{ab} &= \frac{b}{ab} + \frac{a}{ab} - \frac{c}{ab} \\ &= \frac{b+a-c}{ab}\end{aligned}$$

Reciprocal of $x = x^{-1}$

$\therefore$ Reciprocal of $\dfrac{b+a-c}{ab} = \left(\dfrac{b+a-c}{ab}\right)^{-1} = \dfrac{ab}{a+b-c}$

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14. Functions, 2ADV F1 EQ-Bank 29

$$\begin{aligned}\frac{4}{x^2-9} - \frac{2x+1}{x+3} &= \frac{4-(2x+1)(x-3)}{(x+3)(x-3)} \\ &= \frac{4-(2x^2-5x-3)}{(x+3)(x-3)} \\ &= \frac{-(2x^2-5x-7)}{(x+3)(x-3)} \\ &= \frac{-(2x-7)(x+1)}{(x^2-9)}\end{aligned}$$

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15. Functions, 2ADV F1 EQ-Bank 30

$4kx^2 - (3-4k)x + k = 0$

1 root $\Rightarrow \Delta = 0$

$$\begin{aligned}\Delta &= b^2 - 4ac \\ 0 &= [-(3-4k)]^2 - 4 \times 4k \times k \\ 0 &= 9 - 24k + 16k^2 - 16k^2 \\ 24k &= 9 \\ k &= \frac{9}{24} = \frac{3}{8}\end{aligned}$$

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16. Functions, 2ADV F1 EQ-Bank 32

$2x^2 - kx + k - 2 = 0$

$\Delta = b^2 - 4ac = (-k)^2 - 4 \times 2(k-2) = k^2 - 8k + 16$

Real roots: $\Delta \geqslant 0$

$$\begin{aligned}k^2 - 8k + 16 &\geqslant 0 \\ (x-4)^2 &\geqslant 0\end{aligned}$$

$\therefore$ At least one root exists for all k

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17. Functions, 2ADV 2009 HSC 4b

$x^2 - (k+4)x + (k+7) = 0$

Equal roots when $\Delta = 0$:

$$\begin{aligned}
[-(k+4)]^2 - 4(1)(k+7) &= 0 \\
k^2 + 8k + 16 - 4k - 28 &= 0 \\
k^2 + 4k - 12 &= 0 \\
(k+6)(k-2) &= 0 \\
k &= -6 \text{ or } 2
\end{aligned}$$

$\therefore$ Equal roots when $k = -6$ or 2

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18. Functions, 2ADV 2015 HSC 12d

$x^2 - 8x + k = 0$

Real roots when $\Delta \geq 0$:

$$\begin{aligned}
b^2 - 4ac &\geq 0 \\
(-8)^2 - 4 \times 1 \times k &\geq 0 \\
64 - 4k &\geq 0 \\
4k &\leq 64 \\
k &\leq 16
\end{aligned}$$

$\therefore$ Real roots exists when $k \leq 16$

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19. Functions, 2ADV EQ-Bank 37

$$\begin{aligned}
\frac{p+1}{q-q^3} \div \frac{p^3+p^2}{q^2-q} &= \frac{p+1}{q(1-q^2)} \times \frac{q(1-q)}{p^2(p+1)} \\
&= \frac{(1-q)}{(1+q)(1-q)p^2} \\
&= \frac{1}{p^2(1+q)}
\end{aligned}$$
